Optimum Multiuser Detection for MC-CDMA Systems Using Sphere Decoding

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***Abstract* – The Maximum Likelihood joint detection of all users in a Multi-Carrier Code Division Multiple Access (MC-CDMA) system has a prohibitive complexity, growing exponentially with the number of users, when it is performed using an exhaustive search. In this paper, a novel ML multiuser detection algorithm is proposed, whose complexity is a polynomial function of the number of users and is independent of the modulation size. The MC-CDMA system is modelled using a sphere packing lattice representation and an efficient lattice decoder, called the sphere decoder, is applied to jointly detect all users. Simulation results are proposed up to 64 users transmitting 16-QAM symbols.**

I. Introduction

The Multi-Carrier Code Division Multiple Access (MC-CDMA) technique, initially proposed by [1][2][3], efficiently combines the Orthogonal Frequency Division Multiplex (OFDM) modulation and the code division multiple access technique. In Direct Sequence Code Division Multiple Access (DS-CDMA), the signature and the signal of each user are multiplied in the time domain, whereas, in MC-CDMA, the signature and the signal are multiplied in the frequency domain. For each user, the same symbol is transmitted over several sub-carriers and each signature element multiplies the signal on a different sub-carrier. In other words, the transmitted symbol is spread before OFDM modulation. Thanks to the insertion of a guard interval between OFDM symbols, MC-CDMA systems do not experience inter-symbol interference (ISI) and quasi-synchronism may be obtained in the uplink. Furthermore, the OFDM modulation takes advantage of a large frequency diversity. To combat the multiuser interference introduced by the transmission on a multipath channel, various single-user and multi-user detection techniques have been proposed [4]. Among them, the optimum multiuser detection, based on a Maximum Likelihood (ML) exhaustive search [2], has a prohibitive complexity, growing exponentially with the number of users.

We propose a new optimum detection algorithm with low complexity for MC-CDMA systems. This algorithm, called the Sphere Decoding algorithm, originally employed for sphere packing lattice decoding [5], has recently been proposed to simplify the ML multiuser detection in DS-CDMA [6][7]. The proposed algorithm is optimal with respect to the ML criterion. The receiver models the output of the Maximum Ratio Combining (MRC) operation as a multi-dimensional sphere packing lattice point corrupted by an additive noise and applies the lattice Sphere Decoder to jointly detect all users. This algorithm has a complexity growing polynomially with the number of users and independent of the modulation size. Thus, it allows optimum performance even for full-loaded systems using large modulations. The absence of ISI and the synchronism assumption make the MC-CDMA a more suitable system for lattice representation and sphere decoding than the DS-CDMA system.

The paper is organised as follows: Section II describes the synchronous MC-CDMA system and the corresponding lattice representation. In section III, the sphere decoding algorithm is explained for lattice constellations and then applied to an MC-CDMA system in section IV. Simulation results for a downlink MC-CDMA system are presented in section V and compared to the performance of classical sub-optimum detection algorithms. Finally, some conclusions are drawn in section VI.

II. Lattice representation of a synchronous MC-CDMA system

Let us consider a synchronous MC-CDMA system with K users as described in Fig. 1. At time i and for user k, the transmitted symbol $b_k(i)$, taken from a modulation alphabet A of cardinality |A|, is spread by a signature $\mathbf{c}_k = (c_{k1},\ldots,c_{kL})$, which has good cross-correlation properties with other user signatures. In this paper, signatures belong to an orthogonal Walsh-Hadamard set of size L. After spreading of $b_k(i)$, the L obtained chips are transmitted with signal amplitude ω_k on the L different sub-carriers of an OFDM modulation symbol. We denote $s_k(i)$ the modulated signal filtered by a frequency selective multipath channel. After addition of interfering user signals $\sum_{k'\neq k} s_{k'}(i)$ and Additive White Gaussian Noise (AWGN), OFDM demodulation is performed. The channel is assumed non frequency selective on the sub-carrier bandwidth and is thus described by a single complex coefficient $h_{k\ell}(i)$ for each user k and each sub-carrier ℓ. We denote $\mathbf{C}(i)$ the $K \times L$ matrix combining spreading and channel effects for all users:

$$\mathbf{C}(i) = \begin{bmatrix} c_{11}h_{11}(i) & \cdots & c_{1L}h_{1L}(i) \\ \vdots & & \vdots \\ c_{K1}h_{K1}(i) & \cdots & c_{KL}h_{KL}(i) \end{bmatrix} \quad (1)$$

At time i, the received vector $\mathbf{r}(i) = (r_1(i),\ldots,r_L(i))$ may be expressed as

$$\mathbf{r}(i) = \mathbf{b}(i)\mathbf{D}_\omega\mathbf{C}(i) + \eta(i) \quad (2)$$

where vector $\mathbf{b}(i) = (b_1(i),\ldots,b_K(i))$ contains the K transmitted

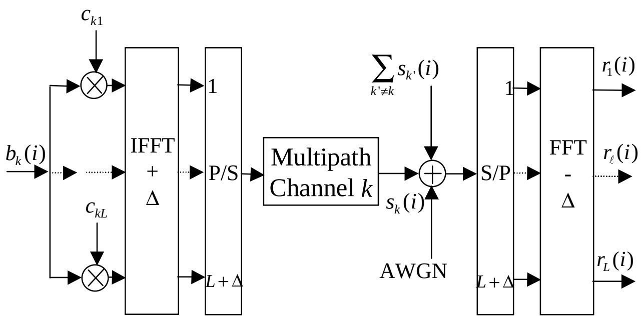

Fig. 1: MC-CDMA transmitter and OFDM receiver.

symbols, diagonal matrix $\mathbf{D}_\omega=\mathrm{diag}(\omega_1,\ldots,\omega_K)$ contains the amplitudes of the different users and $\boldsymbol{\eta}(i) = (\eta_1(i),\ldots,\eta_L(i))$ is the AWGN vector. In downlink, all users share the same channel, defined by $\mathbf{H}(i) = \mathrm{diag}(h_1(i),\ldots,h_L(i))$. Thus, $\mathbf{C}(i) = \mathbf{C}_D\mathbf{H}(i)$ where all user signatures are placed in the $K \times L$ matrix $\mathbf{C}_D = (\mathbf{c}_1^T\ldots\mathbf{c}_K^T)^T$.

Let us now consider the ML multiuser detection using the received signal given in expression (2). In order to maximise the likelihood of the received signal assuming the transmitted signal $\mathbf{b}$, since the noise $\boldsymbol{\eta}(i)$ is AWGN, we have to minimise the quadratic distance $d^2_{\mathrm{Emin}}(\mathbf{b})$ between the received signal and the signal expected to be received if $\mathbf{b}$ has been transmitted: $d^2_{\mathrm{Emin}}(\mathbf{b}) = \|\mathbf{r}(i) - \mathbf{b}\mathbf{D}_\omega\mathbf{C}(i)\|^2$. Equivalently, we may minimise

$$d^2(\mathbf{b}) = \|\mathbf{b}\mathbf{D}_\omega\mathbf{C}(i)\|^2 - 2\mathrm{Re}\langle \mathbf{b}\mathbf{D}_\omega\mathbf{C}(i)\,;\mathbf{r}(i)\rangle\,, \tag{3}$$

where the scalar product is

$$\begin{aligned}\langle \mathbf{b}\mathbf{D}_\omega\mathbf{C}(i)\,;\mathbf{r}(i)\rangle &= \sum_{k=1}^{K}\omega_k b_k^{\,*}\sum_{\ell=1}^{L} c_{k\ell}^{\,*}(i) h_{k\ell}^{\,*}(i) r_\ell(i) \\ &\stackrel{\Delta}{=} \sum_{k=1}^{K}\omega_k b_k^{\,*} y_k(i)\end{aligned} \tag{4}$$

$y_k(i)$ is the MRC output for user k [3]. Vector $\mathbf{y}(i) = (y_1(i),\ldots,y_K(i))$ is a sufficient statistic for the ML detection of transmitted vector $\mathbf{b}(i)$. The observation $\mathbf{y}(i)$ may be written in a matrix form from equation (4):

$$\mathbf{y}(i) \stackrel{\Delta}{=} \mathbf{r}(i)\mathbf{C}^H(i) \tag{5}$$

where $\bullet^H$ denotes the transpose-conjugate. By including expression (2) in expression (5), we obtain $\mathbf{y}(i)$ as a function of the transmitted vector $\mathbf{b}(i)$:

$$\mathbf{y}(i) = \mathbf{b}(i)\mathbf{D}_\omega\mathbf{C}(i)\mathbf{C}^H(i) + \mathbf{n}(i) = \mathbf{b}(i)\mathbf{M}(i) + \mathbf{n}(i) \tag{6}$$

$\mathbf{y}(i)$ in expression (6) can be seen as a point of a complex sphere packing lattice Λ [8] with dimension K and a complex generator matrix $\mathbf{M}(i) = \mathbf{D}_\omega\mathbf{C}(i)\mathbf{C}^H(i)$, corrupted by a noise $\mathbf{n}(i) = (n_1(i),\ldots,n_K(i))$ such that

$$n_k(i) = \sum_{\ell=1}^{L} c_{k\ell}^{\,*}(i) h_{k\ell}^{\,*}(i)\eta_\ell(i) \tag{7}$$

Working with real lattices is more convenient. A sphere packing lattice of $\mathbf{R}^\kappa$ is a discrete subgroup (or a $\mathbf{Z}$-module) with rank κ of $\mathbf{R}^\kappa$. The real space is denoted $\mathbf{R}$ and the integer ring $\mathbf{Z}$. Each point $\mathbf{x}$ of lattice Λ may be written as the linear combination of κ basis vectors: $\mathbf{x} = b_1\mathbf{v}_1 + b_2\mathbf{v}_2 +\ldots+ b_\kappa\mathbf{v}_\kappa$ where $b_k\in \mathbf{Z}$, $\forall k = 1,\ldots,\kappa$. These basis vectors $\mathbf{v}_k$ are the rows of the $\kappa\times\kappa$ lattice *generator matrix* $\mathbf{G}$. Thus,

$$\mathbf{x} = \mathbf{b}\mathbf{G} \text{ where } \mathbf{b} = (b_1,\ldots,b_\kappa) \in \mathbf{Z}^\kappa\,. \tag{8}$$

In order to match with this definition, all previously defined complex vectors (resp. matrices) of size K (resp. $K\times K$) are written as real vectors (resp. matrices) of size $2K$ (resp. $2K\times 2K$). Each complex value a is written as $a = a^R + j.a^I$.

$$\begin{aligned}&\textit{E.g.},\ \mathbf{b}_2(i) = \left(b_1^R(i), b_1^I(i),\ldots,b_K^R(i), b_K^I(i)\right) \\ &\mathbf{D}_{\omega,2} = \mathrm{diag}(\omega_1,\omega_1,\ldots,\omega_K,\omega_K)\end{aligned} \tag{9}$$

$$\mathbf{R}_2(i) = \begin{bmatrix} R_{11}^R & R_{11}^I & \cdots & R_{1K}^R & R_{1K}^I \\ -R_{11}^I & R_{11}^R & \cdots & -R_{1K}^I & R_{1K}^R \\ \vdots & \vdots & & \vdots & \vdots \\ R_{K1}^R & R_{K1}^I & \cdots & R_{KK}^R & R_{KK}^I \\ -R_{K1}^I & R_{K1}^R & \cdots & -R_{KK}^I & R_{KK}^R \end{bmatrix}$$

where $\mathbf{R}(i) = \mathbf{C}(i)\mathbf{C}^H(i)$. With these notations, we obtain

$$\mathbf{y}_2(i) = \mathbf{b}_2(i)\mathbf{M}_2(i) + \mathbf{n}_2(i) \tag{10}$$

where the $2K\times 2K$ matrix $\mathbf{M}_2(i)$ is defined as $\mathbf{M}_2(i) = \mathbf{D}_{\omega,2}\mathbf{R}_2(i)$ and the noise vector $\mathbf{n}_2(i)$ has a covariance matrix $N_0\mathbf{R}_2(i)$. Thus, an MC-CDMA system, with complex modulation and K users, may be modelled as a lattice with dimension $2K$.

If symbols $b_k^R(i)$ and $b_k^I(i)$ belong to a Pulse Amplitude Modulation (PAM) with size M, $(b_k^R(i), b_k^I(i))$ belongs to $(-M + 1, -M + 3,\ldots,M - 3,M - 1)^2$. In this case, to perfectly match the definition of a real lattice, we apply the transformation

$$\mathbf{y}_2'(i) = \frac{1}{2}\left(\mathbf{y}_2(i) + (M-1,\ldots,M-1)\mathbf{M}_2(i)\right) \tag{11}$$

to obtain a vector $\mathbf{b}_2'(i) = (\mathbf{b}_2(i) + (M - 1,\ldots,M - 1)) / 2$, whose elements belong to a segment of $\mathbf{Z}$, such that

$$\mathbf{y}_2'(i) = \mathbf{b}_2'(i)\mathbf{M}_2 + \mathbf{n}_2'(i) \text{ with } \mathbf{b}_2'(i)\in\mathbf{Z}^{2K} \tag{12}$$

For other modulations, an appropriately chosen transformation must be applied to obtain integer coordinates. $\mathbf{y}_2'(i)$ is a point of a lattice Λ_2, with dimension $2K$ and generator matrix $\mathbf{M}_2(i)$ corrupted by an additive noise $\mathbf{n}_2'(i) = \mathbf{n}_2(i) / 2$ with covariance matrix $\mathbf{R}_2(i)N_0 / 4$. If signatures are independent and all amplitudes are greater than zero, Λ_2 is a $\mathbf{Z}$-module with rank $2K$ of the $2K$-dimensional real space $\mathbf{R}^{2K}$. The multiple access system generates a point $\mathbf{b}_2'(i)\mathbf{M}_2(i)$ belonging to a constellation, *i.e.*, a finite subset of Λ_2, with size $|A|^K$.

The MC-CDMA system is convenient for lattice representation since no ISI exists and the frequency selectivity is easily taken into account. However it requires a new computation of the generator matrix each time the channel is modified. The lattice representation allows us to use the Sphere Decoding algorithm [5], a low complexity ML decoding algorithm, capable of decoding any lattice defined by an arbitrary generator matrix $\mathbf{G}$. This algorithm is directly inspired by an algorithm, which, for a given point in the space, finds the closest point in the lattice (*closest vector problem*) [9].

Since the sphere decoding has a complexity in $O(K^6)$ and since the lattice dimension for a complex modulation is twice the lattice dimension for a real modulation, the decoding complexity is 32 times higher per dimension with a complex modulation. The problem is simpler in the special case of a downlink transmission with real spreading, when the in-phase and quadrature signals are spread by the same real sequence, *i.e.*, when matrix $\mathbf{C}_D$ is real valued:

$$\mathbf{M}(i) = \mathbf{D}_\omega \mathbf{C}_D \mathbf{H}(i)\mathbf{H}^H(i)\mathbf{C}_D{}^H = \mathbf{D}_\omega \mathbf{C}_D \left|\mathbf{H}(i)\right|^2 \mathbf{C}_D{}^T \quad (13)$$

where $|\mathbf{H}(i)|^2 = \mathrm{diag}(|h_1|^2,\ldots,|h_L|^2)$ is a real matrix. Hence, the lattice generator matrix $\mathbf{M}(i)$ is real valued and the system may be modelled as a real lattice with dimension K and generator matrix $\mathbf{M}(i)$:

$$\begin{aligned} \mathbf{y}^R(i) &= \mathbf{b}^R(i)\mathbf{M}(i) + \mathbf{n}^R(i) \\ \mathbf{y}^I(i) &= \mathbf{b}^I(i)\mathbf{M}(i) + \mathbf{n}^I(i) \end{aligned} \quad (14)$$

where $\mathbf{y}^R(i)$, $\mathbf{b}^R(i)$, $\mathbf{n}^R(i)$ (resp. $\mathbf{y}^I(i)$, $\mathbf{b}^I(i)$, $\mathbf{n}^I(i)$) are vectors containing real parts (resp. imaginary parts) of the elements of $\mathbf{y}(i)$, $\mathbf{b}(i)$, $\mathbf{n}(i)$. Noise vectors $\mathbf{n}^R(i)$ and $\mathbf{n}^I(i)$ have for covariance matrix $\mathbf{R}(i)N_0 = \mathbf{C}_D|\mathbf{H}(i)|^2\mathbf{C}_D{}^T N_0$. Such a representation for real spreading in downlink does not increase the decoding complexity per dimension when using a complex modulation symbol instead of a real one.

III. Sphere decoding of a lattice corrupted by white Gaussian noise

Let us first describe the ML decoding on AWGN channel of a κ-dimensional lattice Λ generated by a real $\kappa \times \kappa$ matrix $\mathbf{G}$. The decoder has to find the closest lattice point to the received vector, *i.e.*, to minimise

$$m(\mathbf{y}/\mathbf{x}) = \sum_{i=1}^{\kappa} \left|y_i - x_i\right|^2 = \left\|\mathbf{y} - \mathbf{x}\right\|^2 \quad (15)$$

where $\mathbf{y} = \mathbf{x} + \boldsymbol{\eta}$ is the received vector, $\boldsymbol{\eta} = (\eta_1,\ldots,\eta_\kappa)$ the noise vector and $\mathbf{x} = (x_1,\ldots,x_\kappa)$ a point belonging to Λ. The noise vector $\boldsymbol{\eta}$ has real independent elements following a Gaussian distribution with zero mean and variance N_0. Lattice points $\{\mathbf{x} = \mathbf{bG}\}$ are obtained from data vectors $\mathbf{b} = (b_1,\ldots,b_\kappa) \in \mathbf{Z}^\kappa$. In practice, the set of data vectors is limited to a finite alphabet $A_\kappa \subset \mathbf{Z}^\kappa$ and an exhaustive ML decoder searches for the best point $\mathbf{x}$ among all points in the finite constellation. The sphere decoder restricts its computation to the points located inside a sphere with a given radius $\sqrt{C}$ centred on the received point, as shown in Fig. 2. Only lattice points located within the quadratic distance C from the received point are thus taken into account for the metric minimisation (15). The decoder performs the following minimisation:

$$\min_{\mathbf{x}\in\Lambda} \left\|\mathbf{y} - \mathbf{x}\right\| = \min_{\mathbf{w}\in\mathbf{y}-\Lambda} \left\|\mathbf{w}\right\| \quad (16)$$

The above equality shows that the shortest vector $\mathbf{w}$ in the translated set $\mathbf{y} - \Lambda$ must be found. We write the received vector $\mathbf{y} = \boldsymbol{\rho}\mathbf{G}$ and the difference $\mathbf{w} = \boldsymbol{\xi}\mathbf{G}$ with $\boldsymbol{\rho} = (\rho_1,\ldots,\rho_\kappa) \in \mathbf{R}^\kappa$ and $\boldsymbol{\xi} = (\xi_1,\ldots,\xi_\kappa) \in \mathbf{R}^\kappa$. $\mathbf{w}$ is a lattice point, whose coordinates ξ_i are expressed on the translated basis centred on the received point $\mathbf{y}$. Since $\mathbf{w}$ must be located in a sphere with quadratic radius C centred on $\mathbf{y}$, *i.e.*, on $\mathbf{0}$ in the new basis,

$$\left\|\mathbf{w}\right\|^2 = Q(\boldsymbol{\xi}) = \boldsymbol{\xi}\mathbf{G}\mathbf{G}^T\boldsymbol{\xi}^T \le C \quad (17)$$

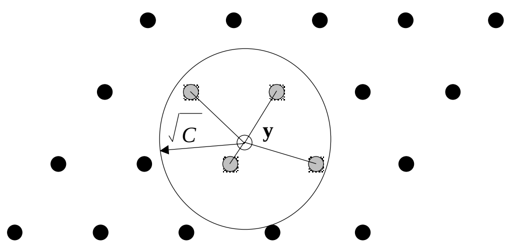

Fig. 2: Geometric representation of the sphere decoding algorithm.

In the new coordinates' system defined by $\boldsymbol{\xi}$, the sphere with squared radius C centred on $\mathbf{y}$ is changed into an ellipse centred at the origin. The Cholesky factorisation of the Gram matrix $\boldsymbol{\Gamma} = \mathbf{G}\mathbf{G}^T$ yields $\boldsymbol{\Gamma} = \mathbf{A}\mathbf{A}^T$, where $\mathbf{A}$ is a lower triangular matrix with elements a_{ij}. Using (17), it was shown [5] that the inequalities in (18), where $q_{ii} = a_{ii}{}^2$ for $i = 1,\ldots,\kappa$ and $q_{ij} = a_{ij} / a_{jj}$ for $j = 1,\ldots,\kappa$, $i = j + 1,\ldots,\kappa$, define the integer components of $\mathbf{b}$ for all points inside the ellipse. $\lceil x \rceil$ is the *ceil* function and $\lfloor x \rfloor$ is the *floor* function. Lower and upper bounds in (18) show that there are κ internal counters, one counter per dimension, in the sphere decoder. Then, values of the various counters just have to be varied inside the bounds. Values of these bounds depend on values of counters. In practice, they may be recursively updated [5]. We thus enumerate all values of vector $\mathbf{b}$ such that the corresponding lattice point $\mathbf{x} = \mathbf{bG}$ is located within the quadratic distance C from the received point.

$$\begin{gathered}
\left\lceil -\sqrt{\frac{C}{q_{\kappa\kappa}}} + \rho_\kappa \right\rceil \le b_\kappa \le \left\lfloor \sqrt{\frac{C}{q_{\kappa\kappa}}} + \rho_\kappa \right\rfloor \\
\left\lceil -\sqrt{\frac{C - q_{\kappa\kappa}\xi_\kappa{}^2}{q_{\kappa-1,\kappa-1}}} + \rho_{\kappa-1} + q_{\kappa,\kappa-1}\xi_\kappa \right\rceil \le b_{\kappa-1} \le \left\lfloor \sqrt{\frac{C - q_{\kappa\kappa}\xi_\kappa{}^2}{q_{\kappa-1,\kappa-1}}} + \rho_{\kappa-1} + q_{\kappa,\kappa-1}\xi_\kappa \right\rfloor \\
\left\lceil -\sqrt{\frac{1}{q_{ii}}\left(C - \sum_{\ell=i+1}^{\kappa} q_{\ell\ell}\left(\xi_\ell + \sum_{j=\ell+1}^{\kappa} q_{j\ell}\xi_j\right)^2\right)} + \rho_i + \sum_{j=i+1}^{\kappa} q_{ji}\xi_j \right\rceil \le b_i \le \left\lfloor \sqrt{\frac{1}{q_{ii}}\left(C - \sum_{\ell=i+1}^{\kappa} q_{\ell\ell}\left(\xi_\ell + \sum_{j=\ell+1}^{\kappa} q_{j\ell}\xi_j\right)^2\right)} + \rho_i + \sum_{j=i+1}^{\kappa} q_{ji}\xi_j \right\rfloor
\end{gathered} \quad (18)$$

Lattice points located outside the considered sphere are never tested. Consequently, the decoding complexity does not depend on the lattice constellation size $|A_\kappa|$. Furthermore, the search inside the sphere is drastically speeded up by updating radius $\sqrt{C}$ with the last computed Euclidean norm $\|\mathbf{w}\|$. Finally, the selected point $\mathbf{x}$ is the point associated to the minimum norm $\|\mathbf{w}\|$.

The search radius $\sqrt{C}$ must be appropriately chosen. Indeed, since the number of points located in the decoding sphere increases with C, a large value for C slows down the algorithm, whereas the search sphere may be empty if C is too small. In order to ensure that the decoder will find at least a lattice point, we have to choose a search radius greater than the covering radius. *E.g.*, it could be chosen equal to the Rogers upper bound [8]:

$$\sqrt{C}^{\kappa} = \left(\kappa\log\kappa + \kappa\log\log\kappa + 5\kappa\right)\times\frac{\left|\det(\mathbf{G})\right|}{V_\kappa} \tag{19}$$

where V_κ is the volume of a sphere with unity radius in real space $\mathbf{R}^\kappa$.

IV. Sphere decoding of a synchronous MC-CDMA system

To jointly detect all users in an MC-CDMA system, the sphere decoding is applied to the $2K$-dimensional corresponding lattice, one time for each received point, *i.e.*, for K users. In case of real spreading in downlink, it is applied to the K-dimensional corresponding lattice, twice for each received point.

The additive noise samples included in the system model of equations (10) and (14) are correlated. The ML lattice decoder has to minimise the following metric (to simplify notations, all '2' indices, 'i' indices and primes will be omitted in the following):

$$m'(\mathbf{y}/\mathbf{x}) = (\mathbf{y}-\mathbf{x})\mathbf{R}^{-1}(\mathbf{y}-\mathbf{x})^T \tag{20}$$

The sphere decoder equations may be easily adapted to the above-described optimisation. This optimisation is equivalent to the ML decoding of a lattice Λ' with generator matrix $\mathbf{M}$ corrupted by a coloured noise $\mathbf{n}$. Nevertheless, we prefer whitening the noise at the output of the MRC bank, in order to use the same decoding procedure as described in section III. The noise whitening operation performed before decoding is similar to the noise whitening operation in equalisation theory. The Cholesky factorisation of the cross-correlation matrix $\mathbf{R}$ gives $\mathbf{R} = \mathbf{W}\mathbf{W}^T$ where $\mathbf{W}$ is a lower triangular matrix. The whitened observation is defined as $\tilde{\mathbf{y}} = \mathbf{y}\mathbf{W}^{T^{-1}}$, which corresponds to a new lattice point $\tilde{\mathbf{x}} = \mathbf{x}\mathbf{W}^{T^{-1}}$. Equation (20) is thus changed into a metric similar to (15). The relation between the lattice point $\tilde{\mathbf{x}}$ and the data vector $\mathbf{b}$ is

$$\tilde{\mathbf{x}} = \mathbf{x}\mathbf{W}^{T^{-1}} = \mathbf{b}\mathbf{M}\mathbf{W}^{T^{-1}} = \mathbf{b}\mathbf{D}_\omega\mathbf{W} \tag{21}$$

Equation (21) shows that after the whitening operation, we must consider a new lattice with generator matrix $\mathbf{G} = \mathbf{D}_\omega\mathbf{W}$ and thus process the new received point $\tilde{\mathbf{y}}$ with the sphere decoder associated to this new matrix. Since $\mathbf{D}_\omega\mathbf{W}$ is already a lower triangular matrix, the Cholesky factorisation preceding the search in the sphere is now useless ($\mathbf{A} = \mathbf{G}$). In other words, the factorisation operation has been moved from the decoder to the whitening operation. The complete receiver structure is described on Fig. 3.

V. Simulation results

The sphere decoding has been tested in downlink on an indoor channel defined in [10]. The channel coefficients are modified for each transmitted symbol. All users have same power. We assume the power control being perfect, *i.e.*, at each time i, the received symbol power is equal to the transmitted symbol power. Each user symbol is spread over $L = 64$ sub-carriers with a real Walsh-Hadamard sequence and the guard interval is 25% of the OFDM symbol period.

Fig. 4 compares the optimum performance of the sphere decoding with the sub-optimum performance of a Parallel Interference Cancellation (PIC) with 2 iterations including single user Minimum Mean Square Error Combining (MMSEC) and hard cancellation. The modulation is QPSK. The gain achieved by ML detection as compared with the PIC detection increases with the number of users. Furthermore, the degradation due to the multiple access interference, for full-load (64 users) and optimum detection, is not greater than 1.2 dB with respect to single-user performance for an average bit error rate (BER) equal to 10^{-3}. Fig. 5 compares various single-user (MRC, Equal Gain Combining (EGC) [3], Orthogonality Restoring Combining (ORC), MMSEC [1]) and multi-user (ML, PIC [2], Global Minimum Mean Square Error (GMMSE) [11]) detection techniques, for half-load (32 users) and 16-QAM modulation. As previously, the degradation of optimum multiuser performance compared to single-user performance is very low. The improvement with respect to the multiuser GMMSE detection is 1.6 dB for a BER equal to 10^{-3}. Finally, Fig. 6 shows that the gap in performance between optimum and sub-optimum detectors is even greater when the number of users increases to 64 for a 16-QAM modulation. With such a system, an exhaustive ML search would have required the computation of 2^{256} metrics to detect each vector $\mathbf{b}$.

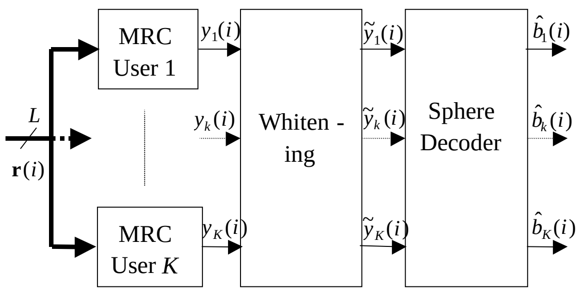

Fig. 3: MC-CDMA receiver with joint detection of the corresponding lattice.

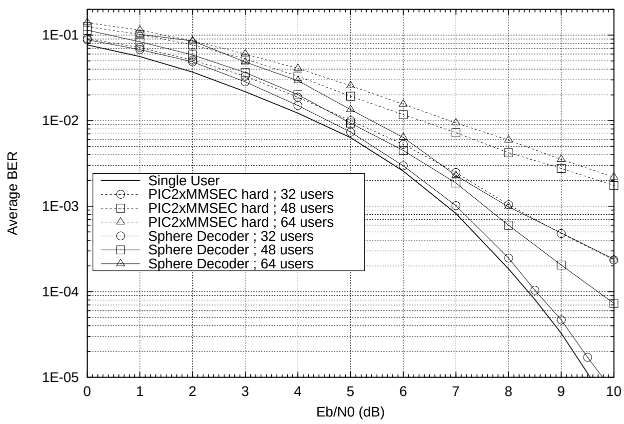

Fig. 4: Comparison between sphere detection and sub-optimum PIC detection, QPSK modulation.

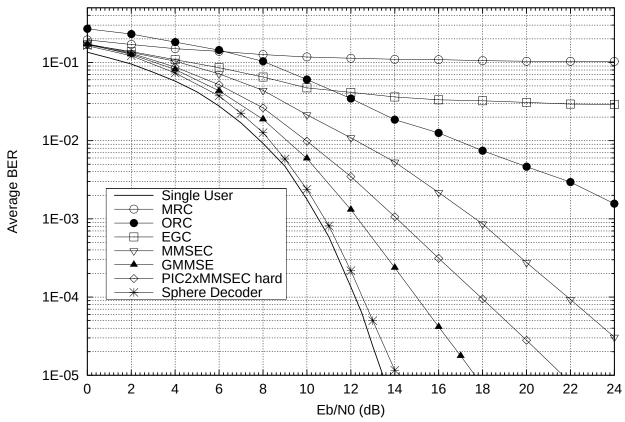

Fig. 5: Comparison of various detection techniques, 32 users, 16-QAM modulation.

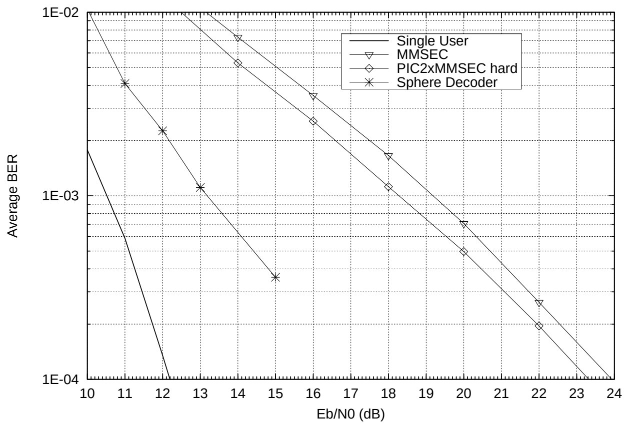

Fig. 6: Comparison of various detection techniques, 64 users, 16-QAM modulation.

VI. Conclusions

We proposed a low complexity optimum multiuser detection for MC-CDMA systems. The received signal is seen as a lattice point corrupted with additive noise and an efficient lattice decoder is applied to detect all users. The complexity of this optimum algorithm is independent of the modulation size and grows polynomially with the number of users. These properties allow us to know performance limits even for high loads and high spectral efficiency modulations. They show us the system intrinsic degradation with respect to single-user performance, which will not be overcome, whatever detection scheme we use, and reveal that sub-optimum detection techniques still do not achieve ML performance limits.

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