# Design of Concatenated Extended Complementary Sequences for inter-base station synchronization in WCDMA TDD mode

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*Abstract* — **The Universal Mobile Telecommunications System (UMTS) being specified by the Third Generation Partnership Project (3GPP) and based upon Wideband Code Division Multiple Access (WCDMA) consists of a frequency division duplex (FDD) mode and a time division duplex (TDD) mode. In the UMTS Terrestrial Radio Access (UTRA) TDD mode, system capacity is maximized with base stations (Node B's) in a UTRA TDD mode deployment area operating synchronously. A Node B synchronization mechanism based upon transmissions and timing measurements of synchronization bursts between neighboring Node B's has been specified as part of release 2000 (now called release 4) in 3GPP. A family of new synchronization codes, so-called Concatenated Extended Complementary (CEC) sequences have been chosen for the Node B synchronization bursts. CEC sequences are derived from Polyphase Complementary Pairs which facilitate the use of efficient and low-complexity receiver implementations. CEC sequences have perfect aperiodic auto-correlation properties within a window of adjustable size and excellent overall auto-correlation properties. This paper describes how to derive CEC sequences and their auto-correlation properties and details the design of CEC sequences adapted to the inter-base station synchronization over the air process.**

*Index terms* — **Third generation mobile radio system, UMTS, WCDMA, TDD mode, inter-base station synchronization, complementary sequences, multiple code offsets**

## I. Introduction

The UTRA TDD mode, part of the third generation mobile radio system proposed by 3GPP to the International Telecommunications Union (in the scope of the IMT 2000 effort) is based on a combined time and code division multiple access scheme. Due to its inherent time division duplex (TDD) component, inter-cell synchronization is necessary for fully exploiting the system capacity. The inter-base station synchronization accuracy to be achieved has been evaluated to be in the order of 3 μs for UTRA TDD.

The most straightforward way of achieving accurate inter-base station synchronization in UTRA TDD is the use of an external timing reference, i.e. GPS. Unfortunately, its use might become unfeasible or unpractical in some deployment scenarios. Therefore, an additional low-cost inter-base station synchronization mechanism, based upon periodical transmissions and timing measurements of special synchronization bursts over the air interface between neighboring base stations (Node B's) has been specified in 3GPP for release 2000 of UTRA TDD.

This paper is organized as follows. Section II briefly introduces the inter-base station synchronization over the air concept, section III reviews complementary sequences from which the CEC sequences are built as described in section IV. Then, in section V, orthogonality preserving families of CEC sequences derived from one constituent pair of complementary sequences are described. Recently adopted CEC sequences in the 3GPP WCDMA TDD standard are finally presented in section VI.

## II. Inter-base station synchronization over the air in WCDMA TDD mode

This additional inter-base station synchronization mechanism aims at synchronizing different UTRA TDD Node B's controlled by one RNC (Radio Network Controller). It is furthermore assumed that at least one of the Node B's controlled by the RNC is equipped by an external timing equipment.

Typically, Node B's in an RNC-area receive scheduling information when to transmit and when to receive special inter-base station synchronization bursts in some reserved uplink random access timeslots. A Node B that receives a synchronization burst, a so-called cell synchronization sequence, can measure and report the corresponding timing offset back to the RNC. Typically, every Node B would itself transmit such a cell synchronization sequence every 10 to 15 seconds and receive and measure cell synchronization sequences from neighboring Node B's once every second. From the timing offset measurements and the reference timing, the RNC can derive the necessary clock corrections for the Node B's it controls. The procedure can be divided in two distinct phases, Initial Synchronization and Steady-State Phase:

- *Initial Synchronization*

At network start up, where no traffic is supported, cells of an RNS (Radio Network Sub-system) area must get synchronized. All cells successively transmit the same cell synchronisation sequence. By measuring transmissions from surrounding cells, each cell can determine and report the timing and received SIR of successfully detected cell synchronisation sequences to the RNC which in turn uses these measurements to adjust the timing of each cell to achieve the required synchronization accuracy. Node B's are coarsely time-aligned with a precision in the order of several radio frames by signaling from their RNC. The cell synchronization sequence is received with a timing uncertainty in the order of several radio frames.

- *Steady-State Phase*

In this phase, traffic is supported by the network and inter-base station synchronization over the air is designed to avoid any interference with existing traffic. Cells of an RNS use the same basic synchronization code but are differentiated by the appliance of different code offsets. The RNC schedules which cells shall transmit and which ones receive and signals to the appropriate cells all transmit parameters including transmission instant, transmission power, code and offsets and the receive parameters (codes and code offsets to measure in a certain timeslot). The maximum clock drift to be corrected and that neighboring Node B's will experience is in the order of several µs. Furthermore, channel delay spread in the order of up to 10-20 µs and propagation delay must be taken into account.

## III. Complementary Sequences

Complementary sequences are known since the late 1940's and have already found major applications in the fields of optics and radar. The key property of such a pair of complementary sequences is that the sum of their aperiodic auto-correlation functions is a perfect Dirac function, e.g. zero for all non-zero time shifts. Thus, with *s(n)* and *g(n)* designating such a pair of complementary sequences of length *N*, their respective aperiodic auto-correlation functions $\varphi_{s,s}(m)$ and $\varphi_{g,g}(m)$ sum up to,

$$\varphi_{s,s}(m) + \varphi_{g,g}(m) = 2\,N\,\delta(m) \qquad (1)$$

Golay complementary sequences can be constructed for any lengths which are an integer power of 2, 10, 26 or any combinations of these. The perfect auto-correlation sum property of (1) is shown for the example of a length $N = 16$ Golay complementary pair in figure 1.

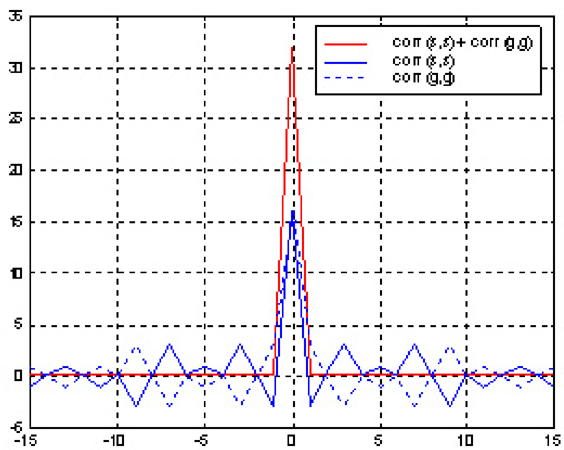


*Figure 1: Auto-correlation sum of length N = 16 Golay complementary sequences*

Golay complementary sequences of integer power of 2 length are a special binary case of more general Polyphase complementary pairs [2] that can be constructed by a recursive relationship. An efficient matched-filter implementation, the Enhanced Golay Correlator (EGC) is known for these Polyphase complementary pairs [3]. An EGC performs a simultaneous correlation with *s(n)* and *g(n)* constituting a Polyphase complementary pair of length *N* with $O(\log_2(N))$ multiplications and additions. This low complexity reduces even more for Golay complementary sequences, as multiplications reduce to simple sign inversions.

## IV. CEC sequences

### A. Design considerations

The cell synchronization sequence can have a length of up to 2400 chips, taking into account an extended guard period of length 160 chips in a timeslot of overall duration 2560 chips (0.666ms). It should have good overall aperiodic auto-correlation properties, especially for small shifts around the main correlation peak, as this region is mainly concerned in the Steady-State phase. In addition, the computational complexity necessary for the correlation process should be kept as low as possible for low cost implementation.

### B. Construction principle

Sequences derived from Polyphase complementary pairs are an attractive choice as synchronization sequences due to their excellent aperiodic auto-correlation properties and existence of low-complexity receiver structures (EGC's). A cyclically extended complementary (CEC) sequence *e(n)* is derived from a pair of constituent Polyphase complementary sequences *s(n)* and *g(n)* by means of cyclic pre and/or post extensions. With $s_{ext}(n)$ and $g_{ext}(n)$ denoting the cyclically extended Polyphase complementary sequences, a CEC sequence *e(n)* is obtained as concatenation of $s_{ext}(n)$ and $g_{ext}(n)$. *N* is an integer power of 2. In figure 2, this construction principle is shown for the case of a CEC sequence derived by means of both pre and post extensions.

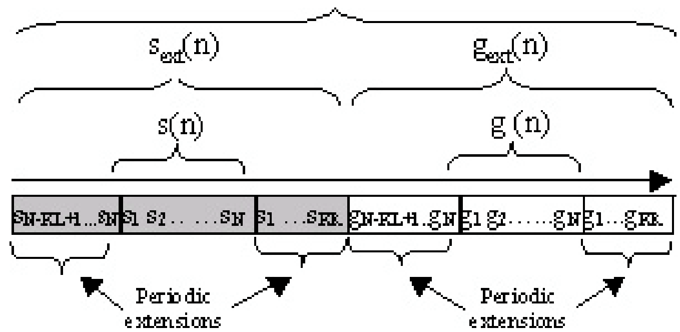


*Figure 2: Construction principle of a CEC sequence with pre and post extensions*

In an alternative way, CEC sequences can also be constructed by leaving out either pre or post extension. An example for a CEC sequence *e(n)* derived with only a post extension is shown in figure 3.

CEC sequences such as shown in figure 2 and 3 are equivalent in terms of their auto-correlation properties. However, generating a family of CEC sequences from only one constituent Polyphase complementary pair (see Section V) is conceptually closer related to a CEC sequence as derived in figure 3.

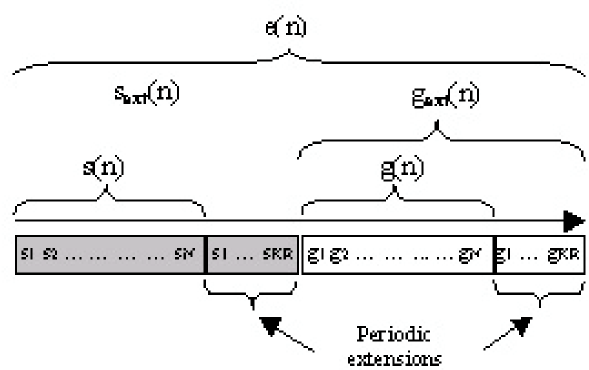

*Figure 3: Construction principle of a CEC sequence with post extensions only*

### C. Correlation properties

With $K_L$ and $K_R$ denoting the length of the pre and post extensions to the constituent Polyphase complementary pairs, the CEC sequence *e(n)* of figure 2 is given by,

$$e(n) = s_{ext}(n) + g_{ext}(n - N - (K_L + K_R)) \qquad (2)$$

Two different ways to perform a correlation with this CEC sequence are possible. In a first approach, the receiver can perform a simple, plain correlation of the received signal with *e(n)*. The corresponding aperiodic auto-correlation function can easily be derived from (2) and generally yields excellent overall correlation properties. The second possible approach exploits the complementary property of the constituent sequences building the CEC sequence *e(n)*. The receiver would,

(A) correlate the received signal with a local replica of *s(n)*,

(B) correlate the received signal at a $N+K_L+K_R$ chip offset with a local replica of *g(n),*

(C) and finally add up the correlation results from (A) and (B) in order to compute an auto-correlation sum.

The correlation of the received signal with *s(n)* and *g(n)* in steps (A) and (B) is efficiently implemented by EGC's. The simplified receiver structure corresponding to this 3-step correlation procedure is illustrated in figure 4.

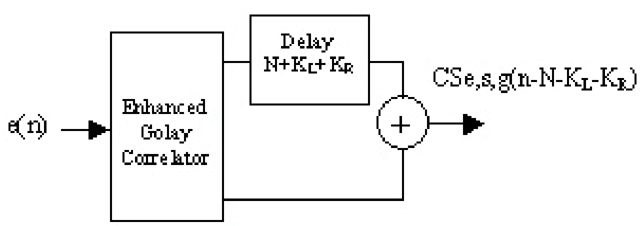


*Figure 4: Simplified receiver structure for 3-step correlation procedure*

The correlation sum $CS_{e,s,g}$, obtained after step (C) of the above correlation procedure,

$$CS_{e,s,g}(n) = \varphi(e(n), s(n)) + \varphi(e(n), g(n - N - (K_L + K_R)) \qquad (3)$$

can be developed into

$$\begin{aligned} CS_{e,s,g}(n) &= \varphi(s_{ext}(n), s(n)) \\ &+ \varphi(g_{ext}(n - N - (K_L + K_R)), g(n - N - (K_L + K_R))) \\ &+ \varphi(s_{ext}(n), g(n - N - (K_L + K_R))) \\ &+ \varphi(g_{ext}(n - N - (K_L + K_R)), s(n)) \end{aligned} \qquad (4)$$

This last expression can be further simplified. More particularly, within [-$K_L$ , $K_R$] around the main correlation sum peak, the correlation sum $CS_{e,s,g}$ in equation (4) can be written as,

$$\varphi(s_{ext}(n), s(n)) + \varphi(g_{ext}(n - N - (K_L + K_R)), g(n - N - (K_L + K_R))) \qquad (5)$$

Taking into account that $s_{ext}(n)$ is a periodically extended version of *s(n)* with left and right extensions of respective lengths $K_L$ and $K_R$, the left side part of (5) is simply,

$$\varphi(s_{ext}(n), s(n))) = \varphi_{s,s}(n - N) + \varphi_{s,s}(n) \qquad (6)$$

and identically the right part of (5) becomes,

$$\begin{aligned} &\varphi(g_{ext}(n - N - (K_L + K_R)), g(n - N - (K_L + K_R)))) \\ &= \varphi(g_{ext}(n), g(n))) \\ &= \varphi_{g,g}(n - N) + \varphi_{g,g}(n) \end{aligned} \qquad (7)$$

Thus by combining (6) and (7), the correlation sum $CS_{e,s,g}$ of equation (4) inside an [-$K_L$ , $K_R$] offset around the main correlation sum peak is obtained as,

$$\begin{aligned} &\varphi(s_{ext}(n), s(n)) + \varphi(g_{ext}(n - N - (K_L + K_R)), g(n - N - (K_L + K_R))) \\ &= 2\delta(n) + 2\delta(n - N) \end{aligned} \qquad (8)$$

The resulting auto-correlation sum from step (C) exhibits a maximum correlation sum peak of $2N$ at the zero time offset and has the very desirable feature not to have any secondary peaks inside the interval $n \in [-K_L, K_R]$ around its main correlation sum peak (figure 5 and 6).

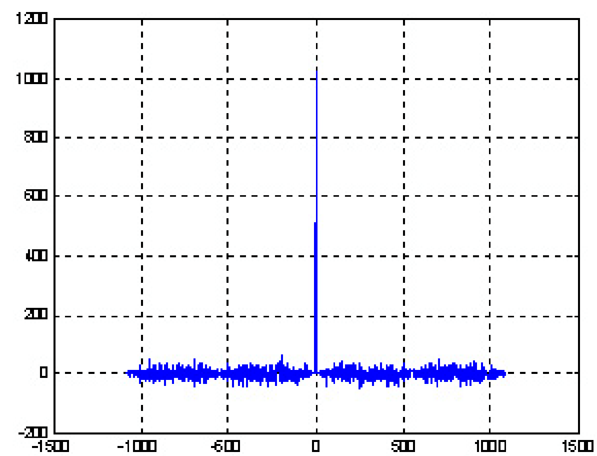


*Figure 5: Example of a CEC sequence correlation sum for $K_L$=$K_R$=20 and N=512*

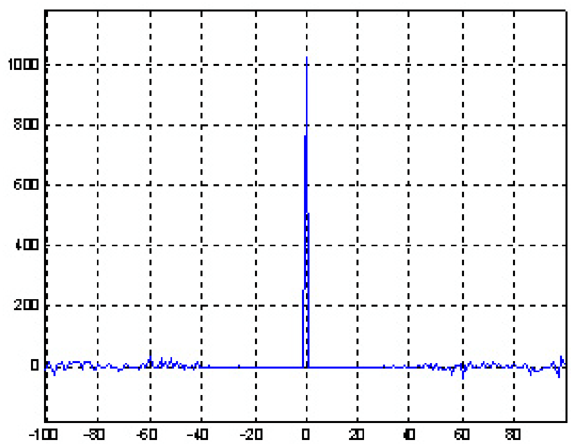


*Figure 6: Example of a CEC sequence correlation sum for $K_L$=$K_R$=20 and N=512 (Zoom)*

CEC sequences as in figure 2 offer the possibility of a perfect auto-correlation window of adjustable size around the main correlation sum peak while maintaining excellent overall auto-correlation properties of either the simple correlation or the correlation sum. The size of the window depends on the length of the periodic extensions.

In the case of CEC sequences constructed with only pre or post extensions (figure 3), an expression for the CEC sequence *e(n)* similar to equation (2) can be derived. Also, the 3-step correlation procedure in order to compute the correlation sum can be performed with only a slight modification.

In step (A), the receiver would correlate the received signal with a cyclically rotated version of *s(n)*, here denoted as *s'(n)*. The elements of *s'(n)* are obtained from the original constituent sequence *s(n)* as being,

$$s_i' = \begin{cases} s_{(K_R-1)/2+i} & i \le N-(K_R-1)/2 \\ s_{(K_R-1)/2+i-N} & i > N-(K_R-1)/2 \end{cases}$$

Here, $K_R$ denotes the length of the post-extension and $N$ the length of the complementary pair. If $K_R$ is an odd number, the nominal code correlation position starts with $s_i$ where $i=(K_R-1)/2+1$.

The receiver would then in step (B) proceed in an analogue way to correlate the received signal with a cyclically rotated version of the constituent sequence *g(n)* at a time offset $N+K_R$ chips and finally add up the corresponding correlation values in order to obtain the auto-correlation sum. When removing the pre extension and correlating with a cyclically rotated version of the constituent sequences, the size of the perfect auto-correlation window around the main correlation sum peak is reduced to $\pm K_R/2$. If $K_R$ is not an odd number, the perfect auto-correlation window becomes very slightly asymmetrical.

## V. CEC-Sequences with Multiple Code Offsets

Several cell synchronization sequences need to be specified in the system for assuring inter-operability between Node B's in a TDD deployment area. It is desirable that this set of cell sync sequences possess very good mutual cross-correlation properties.

With CEC sequences such as derived in section IV, a single cell synchronization sequence is obtained from a particular constituent Polyphase complementary pair. Allowing several Node B's in an area to transmit simultaneously, means to use several of these constituent Polyphase complementary pairs.

Another approach consists in using different code offset versions of the same Polyphase complementary pair in order to differentiate between several Node B's. An advantage is that because of the complementary property, orthogonality is preserved between different cell synchronization sequences derived from the same constituent Polyphase complementary pair by means of a different code offset. It is therefore advantageous to generate an entire family of CEC-sequences from one particular constituent Polyphase complementary pair by allowing variable cyclic shifts of the basic sequences *s(n)* and *g(n)*.

The cyclically shifted versions of *s(n)* and *g(n)*, referred to as $S_m$(n) and $G_m$(n) for code offset $m$ are derived by selecting appropriate elements from the repetitions of *s(n)* and *g(n)* respectively (figure 7). The periodically repeated version of *s(n)* is denoted by $s_e(n)$, with its elements given as $s_{e,i} = \begin{cases} s_i & i \le N \\ s_{i-N} & i > N \end{cases}$.

Then the elements of $S_m$(n), denoted as $S_{m,i}$ are given by $S_{m,i} = s_{e,i+(m-1)W}$, where $W$ is the offset in terms of number of chips. Typically, W is chosen to allow shifts of equal size. With $W$=$K$, the total available number of offsets $M$ is then given by $M$=$N/K$. The corresponding cyclically shifted versions of $G_m$(n) are constructed in identical fashion.

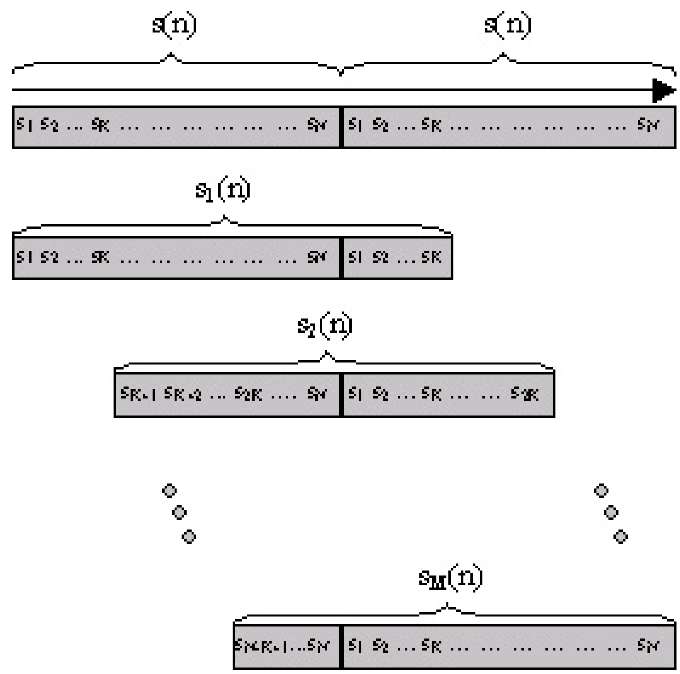


*Figure 7: Deriving different code offset versions of the constituent sequence s(n)*

The overall cell synchronization sequence derived from a particular Polyphase complementary pair *s(n)* and *g(n)* and corresponding to a particular code offset $m$ is finally given by the concatenation of $S_m$(n) and $G_m$(n) as illustrated in figure 8. Cell synchronization sequences build from CEC sequences with multiple code offsets have an overall length of *2(N+K)* chips.

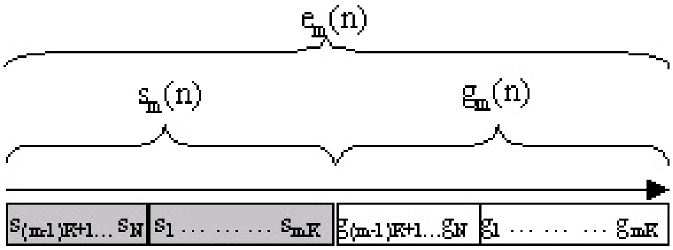


*Figure 8: Cell synchronization sequence derived from a constituent pair s(n) and g(n) for code offset m*

The receiver in a first step cyclically correlates the first half of the overall received signal with a local replica *s(n)*. The first *K* chips of the *N+K* chip long segment corresponding to *s(n)* are discarded. This is equivalent to the computation of the periodic auto-correlation with the local replica *s(n)* by means of a cyclic shift register.

The correlation with *g(n)* is done in a second step in an analogue manner on the second half of the overall received signal. By discarding the first *K* chips, any undesired cross-correlation between the parts corresponding to *s(n)* and *g(n)* due to a multi-path channel with channel impulse response length smaller than *K* can be avoided. Finally, the auto-correlation sum is obtained after adding up corresponding matched-filter outputs from the first and second step.

A typical auto-correlation obtained for the case of *N=1024* and *M=8* possible code offsets with a resolution of *K=128* chips between different simultaneously transmitting Node B's is shown in figure 9 for the case of code offsets *1, 3* and *7* being present.

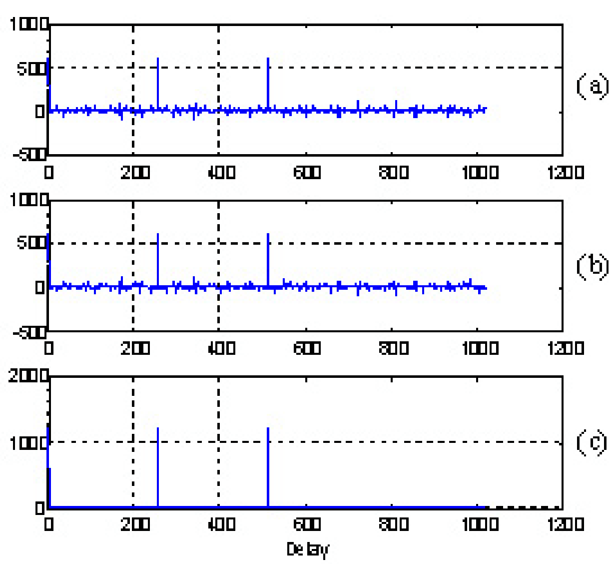


*Figure 9: Example for correlation sum of a CEC sequence with offsets 1, 3 and 7 present for N=1024, K=64 chips, (a) first half, (b) second half, (c) sum*

## VI. CEC-Sequences for WCDMA-TDD

Golay complementary pairs of length *N=1024* have been chosen for building the CEC-sequences with multiple offsets, as the EGC receiver structure simplifies the most for this special binary case of Polyphase complementary sequences. Providing *K=8* possible code offsets for one RNS area leaves *W=128* chips of resolution between different Node B's which is enough to satisfy the design constraints with respect to maximum propagation delay and delay spread as mentionned in Section IV. The overall length of a cell sync sequence is therefore *2176* chips which yields almost a maximum usage of the available time in the cell sync timeslots.

Orthogonality is ensured for up to 8 Node B's transmitting their respective cell sync bursts simultaneously by means of *different code offsets* based on the same constituent Golay complementary pair.

Under some operation conditions however, the coarse time alignment between Node B's, which is a pre-requisite for the use of different code offsets is not possible. Situations like this could include Initial Synchronization and also that of Node B's in a deployment area belonging to different operators. In these cases, different cell synchronization codes can also be derived from up to *8 different constituent Golay complementary pairs.* These 8 constituent Golay complementary pairs have been chosen upon their overall correlation properties. Their generating delay and weight matrices are listed in Table 1, the Code ID *c* corresponding to the 8 possible color codes of a Node B. The delay and weight matrices are used to generate the Golay complementary codes by means of a recursive relationship [2]. CEC sequences with multiple code offsets based on Golay complementary pairs being binary sequences, a continuously increasing Pi/2 phase offset is applied in order to decrease the peak-to-average ratio. All correlation properties remain invariant under this phase rotation.

| *c* | **Delay matrices $D_m$** | **Weight matrices $W_m$** |
|---|---|---|
| 0 | 512,64,128,1,16,4,256,32,8,2, | 1, 1, 1, 1, -1, -1, 1, 1, 1, 1 |
| 1 | 2,16,32,256,1,8,128,4,512,64 | 1, -1, 1, -1, 1, -1, -1, 1, -1, -1 |
| 2 | 16,512,32,256,4,1,64,8,2,128 | -1, 1, 1, -1, -1, 1, -1, 1, -1, -1 |
| 3 | 512,16,8,4,2,256,128,64,32,1, | -1, -1, -1, -1, -1, 1, -1, 1, 1, 1 |
| 4 | 512,128,256,32,2,4,64,1,16,8, | 1, -1, 1, -1, -1, -1, -1, -1, -1, 1 |
| 5 | 1,2,4,64,512,16,32,256,128,8, | -1, 1, 1, 1, 1, -1, -1, 1, -1, 1 |
| 6 | 8,16,128,2,32,1,256,512,4,64, | -1, -1, 1, 1, 1, 1, -1, -1, -1, 1 |
| 7 | 1,2,128,16,256,32,8,512,64,4, | 1, 1, -1, -1, -1, -1, 1, -1, -1, -1 |

*Table 1: Constituent Golay complementary pairs in WCDMA TDD mode*

## VII. Conclusion

In this paper, the inter-base station synchronization codes for the UTRA WCDMA TDD mode have been presented. These codes are constructed as CEC sequences with multiple code offsets, providing very good overall correlation properties, perfect channel estimation windows of adjustable size while enabling the use of low complexity receiver structures. Due to the complementary property of CEC-sequences, their code offset versions remain orthogonal, e.g. without any undesired cross-correlation which makes them a very attractive choice for synchronization purposes.

## VIII. ACKNOWLEDGEMENTS

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