Concatenated Extended Complementary Sequences for inter-base station synchronization in UMTS TDD mode

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***Abstract* — This paper presents a new family of synchronisation sequences, Concatenated Extended Complementary (CEC) sequences. CEC-sequences have recently been adopted for Release 4 of UMTS TDD mode as inter-base station synchronisation codes. They provide perfect auto-correlation properties within a window of adjustable size, excellent overall auto-correlation properties and allow the use of low-complexity receiver structures. These features make CEC-sequences an attractive choice for synchronisation or channel estimation purposes.**

I. Introduction

This paper describes how to derive CEC sequences from their parent polyphase complementary pairs and details their correlation properties. Several possible receivers are described and the design of CEC sequences with multiple offsets preserving orthogonality as adopted for the inter-base station synchronization over the air process in UMTS TDD mode are also described. Section II briefly introduces complementary sequences from which the CEC sequences are built as described in section III. Then, in section IV, orthogonality preserving families of CEC sequences derived from one constituent pair of complementary sequences are described. Recently adopted CEC sequences in the 3GPP WCDMA TDD standard are finally presented in section V.

II. Complementary Sequences

Complementary sequences are known since the late 1940's and have already found major applications in the fields of optics and radar. The key property of such a pair of complementary sequences is that the sum of their aperiodic auto-correlation functions is a perfect Dirac function, e.g. zero for all non-zero time shifts. Thus, with *s(n)* and *g(n)* designating such a pair of complementary sequences of length *N*, their respective aperiodic auto-correlation functions $\varphi_{s,s}(m)$ and $\varphi_{g,g}(m)$ sum up to,

$$\varphi_{s,s}(m) + \varphi_{g,g}(m) = 2\,N\,\delta(m) \qquad (1)$$

Golay complementary sequences can be constructed for any lengths which are an integer power of 2, 10, 26 or any combinations of these. The perfect auto-correlation sum property of (1) is shown for the example of a length $N = 16$ Golay complementary pair in figure 1.

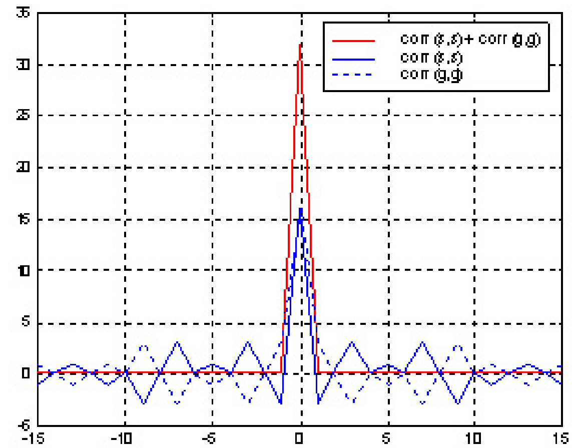

Figure 1: Auto-correlation sum of length N = 16 Golay complementary sequences

Golay complementary sequences of integer power of 2 length are a special binary case of more general Polyphase complementary pairs [2] that can be constructed by a recursive relationship. An efficient matched-filter implementation, the Enhanced Golay Correlator (EGC) is known for these Polyphase complementary pairs [3]. An EGC performs a simultaneous correlation with *s(n)* and *g(n)* constituting a Polyphase complementary pair of length *N* with $O(\log_2(N))$ multiplications and additions. This low complexity reduces even more for Golay complementary sequences, as multiplications reduce to simple sign inversions.

III. CEC sequences

A. Construction principle

Based on the use of Polyphase complementary pairs, CEC-sequences exploit the complementary auto-correlation properties of their constituents sequences. A CEC-sequence *e(n)* is derived from a constituent pair of complementary sequences *s(n)* and *g(n)* of length *N* chips by means of cyclic pre and/or post extensions. With $s_{ext}(n)$ and $g_{ext}(n)$ denoting the cyclically extended complementary pair, a CEC sequence *e(n)* is then obtained by concatenating $s_{ext}(n)$ and $g_{ext}(n)$ (figure 1). K_L and K_R being the length of the pre and post extensions to the constituent Golay complementary pairs.

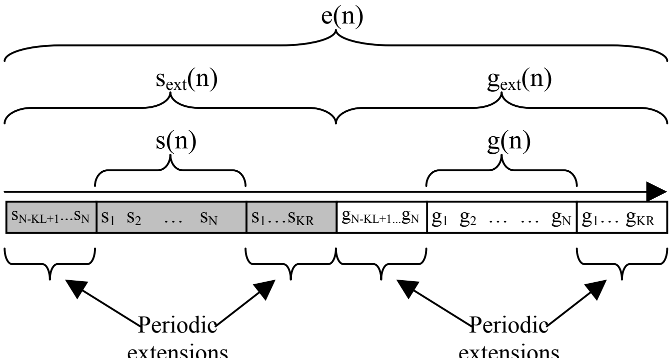

Figure 2: Construction principle of a CEC sequence with pre and post extensions

In an alternative way, CEC sequences can also be constructed by leaving out either pre or post extension. An example for a CEC sequence *e(n)* derived with only a post extension is shown in figure 3.

CEC sequences such as shown in figure 2 and 3 are equivalent in terms of their auto-correlation properties. However, generating a family of CEC sequences from only one constituent Polyphase complementary pair (see Section IV) is conceptually closely related to a CEC sequence as derived in figure 3.

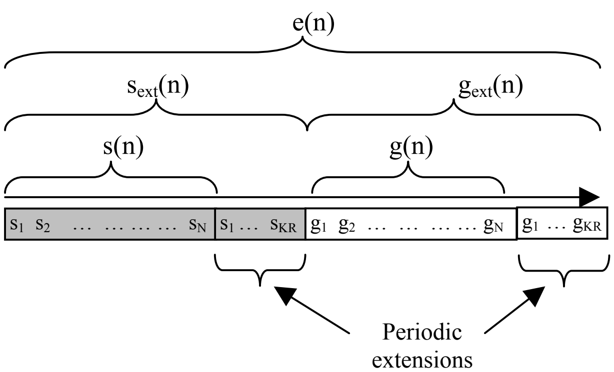

Figure 3: Construction principle of a CEC sequence with post extensions only

B. Correlation properties

With K_L and K_R denoting the length of the pre and post extensions to the constituent Polyphase complementary pairs, the CEC sequence *e(n)* of figure 2 is given by,

$$e(n) = s_{ext}(n) + g_{ext}(n - N - (K_L + K_R)) \qquad n \in \left[-K_L \ , N + K_R\right] \tag{2}$$

CEC sequences correlation properties can be fully exploited, if instead of a simple plain correlation with the overall sequence *e(n)*, the receiver performs a partial correlations with each of its constituent sequences. It first correlates the received signal with a local replica of *s(n)*, and then correlates the received signal at a $N+(K_L + K_R)$ chips offset with a local replica of *g(n)*. In a third and last step, the sum of corresponding correlation results obtained during the previous 2 steps is computed. This 3 step operation is equivalent to a correlation with $e(n) = s(n) + g(n - N - (K_L + K_R))$.

The correlation sum $CS_{e,s,g}(\tau)$ obtained after the third step of the correlation procedure can be written as ,

$$CS_{e,s,g}(\tau) = \varphi_{(e(n),s(n))}(\tau) + \varphi_{(e(n),g(n-N-(K_L+K_R))}(\tau) \tag{3}$$

and can be developed into a sum of correlation terms.

$$\begin{aligned} CS_{e,s,g}(\tau) &= \varphi_{(s_{ext}(n),s(n))}(\tau) \\ &+ \varphi_{(g_{ext}(n-N-(K_L+K_R)),g(n-N-(K_L+K_R)))}(\tau) \\ &+ \varphi_{(s_{ext}(n),g(n-N-(K_L+K_R)))}(\tau) \\ &+ \varphi_{(g_{ext}(n-N-(K_L+K_R)),s(n))}(\tau) \end{aligned} \tag{4}$$

Allowing for the finite duration of the correlation terms and for their different delay offsets, it appears that the global expression can be simplified within $[-K_L, K_R]$ in

$$\varphi_{(s_{ext}(n),s(n))}(\tau) + \varphi_{(g_{ext}(n-N-K_L+K_R),g(n-N-(K_L+K_R)))}(\tau) \qquad for\ \tau \in [-K_L, K_R] \tag{5}$$

Taking into account that $s_{ext}(n)$ [resp $g_{ext}(n)$] is a periodically extended version of s(n) [resp g(n)] with pre and/or post extension of length K_L and/or K_R, the correlation terms between original and extended sequences s(n) and $s_{ext}(n)$ [resp g(n) and $g_{ext}(n)$] can be expressed within $[-K_L, K_R]$ as a sum of 3 auto correlations:

$$\begin{aligned} &\varphi_{(s_{ext}(n),s(n)))}(\tau) = \varphi_{s_{ext},s}(\tau) \\ &= \varphi_{s,s}(\tau - N) + \varphi_{s,s}(\tau) + \varphi_{s,s}(\tau + N) \qquad for\ \tau \in [-K_L, K_R] \end{aligned} \tag{6}$$

$$\begin{aligned} &\varphi_{(g_{ext}(n-N-(K_L+K_R)),g(n-N-(K_L+K_R))))}(\tau) \\ &= \varphi_{(g_{ext}(n),g(n)))}(\tau) \\ &= \varphi_{g,g}(\tau - N) + \varphi_{g,g}(\tau) + \varphi_{g,g}(\tau + N) \ for \ \ \tau \in [-K_L, K_R] \end{aligned} \tag{7}$$

Finally, by replacing (6) and (7) inside (5) the expression within $[-K_L, K_R]$ is reduced to a sum of Dirac:

$$\begin{aligned} &\varphi_{(s_{ext}(n),s(n))}(\tau) + \varphi_{(g_{ext}(n-N-(K_L+K_R)),g(n-N-(K_L+K_R)))}(\tau) \\ &\quad = \varphi_{s,s}(\tau - N) + \varphi_{g,g}(\tau - N) + \varphi_{s,s}(\tau) + \varphi_{g,g}(\tau) \\ &\quad + \varphi_{s,s}(\tau + N) + \varphi_{g,g}(\tau + N) \\ &\quad = 2N\delta(\tau - N) + 2N\delta(\tau) + 2N\delta(\tau + N) \end{aligned} \tag{8}$$

Providing that K_L and K_R values are chosen smaller than N by construction expression (8) is reduced to

$$\varphi_{(s_{ext}(n),s(n))}(\tau)+\varphi_{(g_{ext}(n-N-(K_L+K_R)),g(n-N-(K_L+K_R)))}(\tau) = 2N\delta(\tau) \quad (9)$$

The resulting correlation sum from the third step exhibits a maximum correlation sum peak of $2N$ (for a N bits long pair of Golay parent codes) at the zero time offset and has the very desirable feature not to have any secondary peaks inside the interval $\tau \in [-K_L, K_R]$ around its main correlation sum peak (figure 4 and 5).

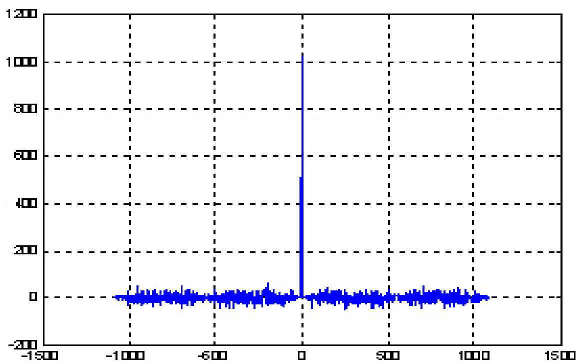

Figure 4: Example of a CEC sequence correlation sum for K_L=K_R=20 and N=512

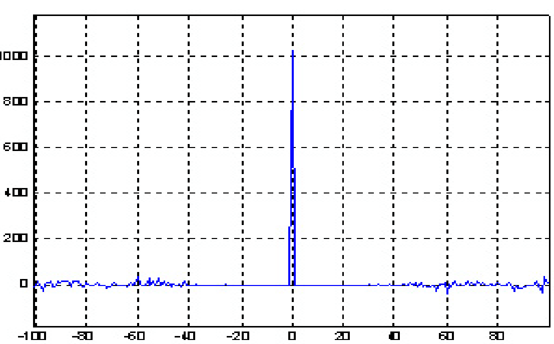

Figure 5: Example of a CEC sequence correlation sum for K_L=K_R=20 and N=512 (Zoom)

CEC sequences offer the possibility of a perfect correlation window of adjustable size around the main correlation sum peak (figure 5) while maintaining excellent overall correlation properties of the correlation sum (figure 6). The size of the window depends on the length of the periodic extensions.

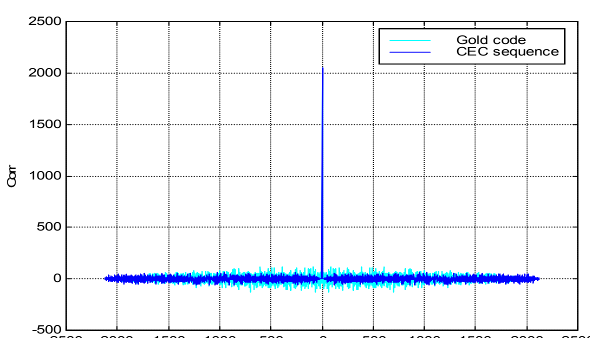

Figure 6: comparison of typical CEC correlation sum and Gold sequences auto correlation (Gold code from octal generators [4005] and [7335] and CEC sequence with K_L=K_R=25 and N=1024)

In the case of CEC sequences constructed with K_R = 0 or K_L=0 (figure 3), all the previous results stay valid and the size of the perfect auto-correlation window around the main correlation sum peak is reduced to $\pm K_L/2$ or $\pm K_R/2$. If K_L *or* K_R is not an odd number, the perfect auto-correlation window becomes very slightly asymmetrical

C. *Receivers*

Elementary 3-step receiver :

The direct application of the correlation sum based respectively on the correlation with the complementary sequences s(n) et g(n) allowing for the appropriate delay leads to the following CEC sequence receiver based on 2 partial and alternate correlations represented on figure 7.

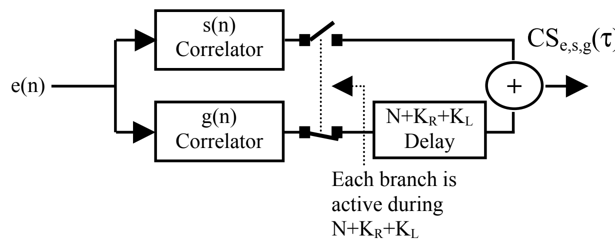

Figure 7 : 3-step correlation receiver

Low complexity Receiver:

The partial correlations with the constituent sequences *s(n)* and *g(n)* are efficiently implemented by an Enhanced Golay Correlator (EGC's) with a related complexity of $O(\log_2(N))$. Correlation complexity can even be more reduced when a Golay complementary pair with a length of N=2^k is chosen as constituent pair.

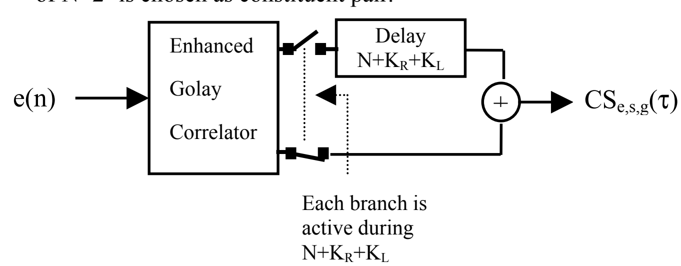

Figure 8: EGC based receiver structure for 3-step correlation procedure

IV. CEC-Sequences with Multiple Code Offsets

A set of CEC sequences can be generated from different offset versions of the same complementary code pair. Orthogonality is preserved between different CEC sequences derived from the same constituent Polyphase complementary pair by means of a different code offset. However a coarse time alignment between tranmitters and receiver is a pre-requisite for the use of different code offsets. Under this condition, it is advantageous to generate an entire family of CEC-sequences from one particular constituent Polyphase complementary pair by allowing variable cyclic shifts of the basic sequences *s(n)* and *g(n)*.

A. Construction principle

The cyclically shifted versions of *s(n)* [resp *g(n)*], referred to as $s_m(n)$ [resp $g_m(n)$] for code offset *m* are derived by selecting appropriate elements from $s_e(n)$ [resp $g_e(n)$] which is the periodic repetition of *s(n)* [resp *g(n)*]

defined by $$s_e(i) = \begin{cases} s(i) & 1 \le i \le N \\ s(i-N) & N < i < 2N \end{cases}$$

[resp $$g_e(i) = \begin{cases} g(i) & 1 \le i \le N \\ g(i-N) & N < i < 2N \end{cases}$$] (c.f. figure 9).

Hence, the elements of $s_m(n)$, are given by $s_m(i) = s_e(i + (m-1)W) \quad i \in [1, N+K]$, where *W* is the offset in terms of number of chips. Typically, W is chosen to allow shifts of equal size. With *W=K*, the total available number M of offsets is then given by *M=N/K*. The corresponding cyclically shifted versions of *g*(n) are constructed in identical way.

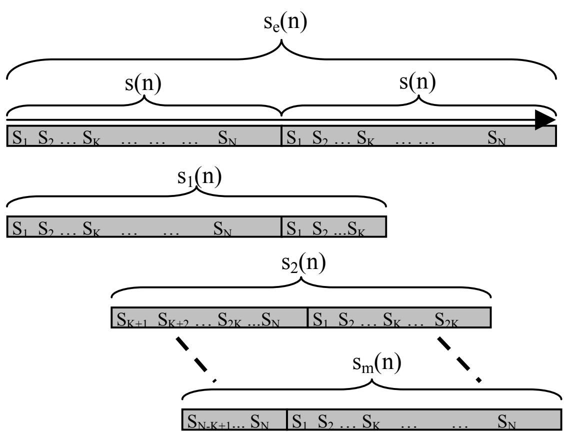

Figure 9: Deriving different code offset versions of the constituent sequence s(n)

The overall cell synchronization sequence derived from a particular Polyphase complementary pair *s(n)* and *g(n)* and corresponding to a particular code offset *m* is finally given by the concatenation of $s_m(n)$ and $g_m(n)$ as illustrated in figure 10. Cell synchronization sequences build from CEC sequences with multiple code offsets have an overall length of *2(N+K)* chips.

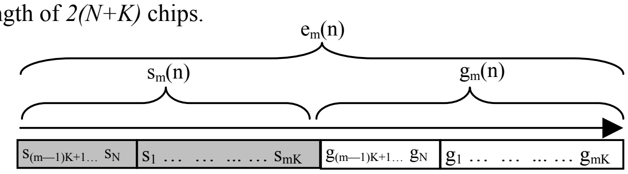

Figure 10: Cell synchronization sequence derived from a constituent pair s(n) and g(n) for code offset m

B. Correlation properties

The perfect correlation properties of CEC sequences with multiple offsets can only be exploited if a pre-requisite coarse time alignment exists at the receiver with a minimum accuracy of K chips.

In a first step, the receiver processes the first half of the overall received signal, corresponding to $s_m(n)$: it discards the first *K* received chips prior to cyclically correlating the remaining N chip long sequence with a local replica *s(n)*. The operation is equivalent to the cyclic correlation computation of a cyclically shifted version of *s(n)* (denoted $s^l(n)$ for a (l-1)W chips cyclic shift) with the local replica of *s(n)* by means of a cyclic shift register.

The correlation with *g(n)* is done in a second step in an analogue manner on the second half of the overall received signal.

By discarding the first *K* chips, any undesired cross-correlation between the parts corresponding to *s(n)* and *g(n)* due to a multi-path channel with channel impulse response length smaller than K can be avoided.

For illustration purpose, let us consider a case where 3 different code offset versions with respective offsets (l-1)W, (m-1)W and (u-1)W are transmitted simultaneously. The cyclic correlation sum obtained after adding up the corresponding correlator outputs from the first and second steps within the delay range [1 , K] is

$$\begin{aligned} CCS_{e,s,g}(\tau) &= \phi_{(s^l(n),s(n))}(\tau) + \phi_{(g^l(n),g(n))}(\tau) \\ &+ \phi_{(s^m(n),s(n))}(\tau) + \phi_{(g^m(n),g(n))}(\tau) \\ &+ \phi_{(s^u(n),s(n))}(\tau) + \phi_{(g^u(n),g(n))}(\tau) \end{aligned} \tag{10}$$

where $\phi(\ )$ denotes cyclic correlation. Cyclic correlation of 2 N-chip long sequences can be decomposed into 2 aperiodic correlation sums

$$\phi_{s,s}(\tau) = \varphi_{s,s}(\tau) + \varphi_{s,s}(\tau - N) \quad \text{and} \tag{11}$$
$$\phi_{g,g}(\tau) = \varphi_{g,g}(\tau) + \varphi_{g,g}(\tau - N)$$

which are equivalent within [1,K] to the aperiodic correlation of s(n) [resp g(n)] with its cyclically extended version with a N chips pre or post extension only. By replacing the cyclic correlation terms inside (10), it gives

$$\begin{aligned} CCS_{e,s,g}(n) &= 2N\,[\delta(n-(l-1)W) + \delta(n-(m-1)W) \\ &+ \delta(n-(u-1)W) + \delta(n-(l-1)W-N) \\ &+ \delta(n-(m-1)W-N) + \delta(n-(u-1)W-N)] \end{aligned} \tag{12}$$

which can be simplified in (13) since τ-N and τ+N are outside the delay observation window.

$$\begin{aligned} &CCS_{e,s,g}(n) = \\ &\quad 2N\,[\delta(n-(l-1)W) + \delta(n-(m-1)W) + \delta(n-(u-1)W)] \end{aligned} \tag{13}$$

The cyclic correlation sum generated a Dirac surrounded by a perfect correlation window of size W for each offset version of the CEC sequence without any inter correlation peak from the other versions.

Figure 11 presents the receiver for CEC sequences with multiple offsets.

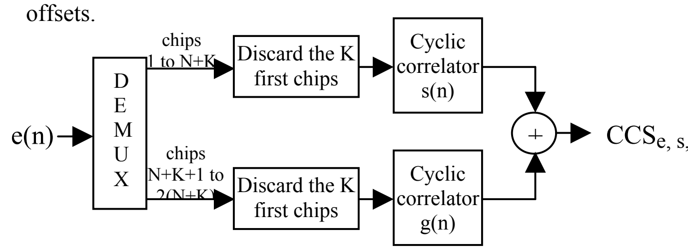

Figure 11: Correlation receiver for CEC sequences with multiple offsets

A typical correlation sum obtained for the case of *N=1024* and *M=8* possible code offsets with a resolution of *K=128* chips per transmitted code offset is shown in figure 12 for the case of code offsets *1, 3* and *5*

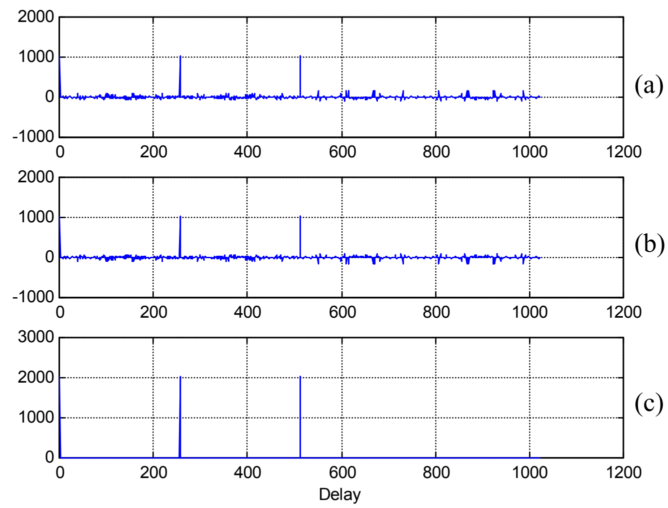

Figure 12: Example for cyclic correlation sum of a CEC sequence with offsets 1, 3 and 5 present for N=1024, K=128 chips, (a) first half, (b) second half, (c) sum

V. CEC-Sequences for WCDMA-TDD

The inter-base station synchronization mechanism aims at synchronizing different UTRA TDD Node B's controlled by one RNC (Radio Network Controller) on a given timing reference.

Upon RNC scheduling, inter-base station synchronization bursts are transmitted in some reserved uplink random access timeslots. A Node B that receives a synchronization burst, a so-called cell synchronization sequence, can measure and report the corresponding timing offset back to the RNC which can use it to derive the necessary clock corrections for the Node B's it controls. The procedure can be divided in two distinct phases, Initial Synchronization and Steady-State Phase:

- *Initial Synchronization*

At network start up, where no traffic is supported, all cells of an RNS (Radio Network Sub-system) area successively transmit **the same cell synchronisation sequence**. By measuring transmissions from surrounding cells, each cell can determine and report the timing and received SIR of successfully detected cell synchronisation sequences to the RNC.

- *Steady-State Phase*

In this phase, Node B's are already coarsely time-aligned, allowing the use of multiple offsets CEC sequences, and traffic is supported by the network. Upon RNC scheduling, cells of an RNS transmit **the same basic synchronization code with dedicated code offsets**.

The cell synchronization sequence can have a length of up to 2400 chips, taking into account an extended guard period of length 160 chips in a timeslot of overall duration 2560 chips (0.666ms).

Golay complementary pairs of length *N=1024* have been chosen for building the CEC-sequences with multiple offsets, as the EGC receiver structure simplifies the most for this special binary case of Polyphase complementary sequences. Providing *K=8* possible code offsets for one RNS area leaves *W=128* chips of resolution between different Node B's which is enough to satisfy the design constraints with respect to maximum propagation delay. The overall length of a cell sync sequence is therefore *2176* chips which yields almost a maximum usage of the available time in the cell sync timeslots.

Under some operation conditions, such as Initial Synchronization or multi operator deployment, the coarse time alignment between Node B's, which is a pre-requisite for the use of different code offsets is not possible. Different cell synchronization codes can also then be derived from up to *8 different constituent Golay complementary pairs* chosen upon their overall correlation properties. Their generating delay and weight matrices are listed in Table 1. The delay and weight matrices are used to generate the Golay complementary codes by means of a recursive relationship [2].

c	**Delay matrices D_m**	**Weight matrices W_m**
0	512,64,128,1,16,4,256,32,8,2,	1, 1, 1, 1, -1, -1, 1, 1, 1, 1
1	2,16,32,256,1,8,128,4,512,64	1, -1, 1, -1, 1, -1, -1, 1, -1, -1
2	16,512,32,256,4,1,64,8,2,128	-1, 1, 1, -1, -1, 1, -1, 1, -1, -1
3	512,16,8,4,2,256,128,64,32,1,	-1, -1, -1, -1, -1, 1, -1, 1, 1, 1
4	512,128,256,32,2,4,64,1,16,8,	1, -1, 1, -1, -1, -1, -1, -1, -1, 1
5	1,2,4,64,512,16,32,256,128,8,	-1, 1, 1, 1, 1, -1, -1, 1, -1, 1
6	8,16,128,2,32,1,256,512,4,64,	-1, -1, 1, 1, 1, 1, -1, -1, -1, 1
7	1,2,128,16,256,32,8,512,64,4,	1, 1, -1, -1, -1, -1, 1, -1, -1, -1

TABLE 1
Constituent Golay Complementary Pairs in WCDMA TDD mode

VI. Conclusion

In this paper, the Concatenated Extended Complementary sequences have been introduced. They provide very good overall correlation properties and perfect channel estimation windows of adjustable size. Their construction principles, correlation properties and several receivers have been detailed. A way of generating CEC sequences with multiple code offsets, whose code offset versions remain orthogonal has also been described. These ones have been adopted for inter-base station synchronization codes in the UTRA WCDMA TDD mode.

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