

Influence of System Load on Channel Estimation in MC-CDMA Mobile Radio Communication Systems

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Abstract

This paper investigates the influence of a variable system load on the performance of channel estimation in a Multicarrier Code Division Multiple Access (MC-CDMA) transmission scheme. We consider channel estimation based on pilot symbols, which are periodically inserted into the MC-CDMA frame. Starting from the analytical expression of the decision variable, we show the impact of erroneous channel estimation as a function of system load. We derive a simple adaptation rule for the pilot symbol power in order to maintain the Signal-to-Noise Ratio (SNR) loss due to channel estimation errors constant for all system loads. We show that this simple rule enables us to minimize the total transmitted power required at a given Bit Error Rate (BER), particularly for systems with high spreading gain as used in mobile radio channels with high frequency diversity.

I. INTRODUCTION

In the research area of transmission techniques for mobile radio systems beyond the 3rd generation, the MC-CDMA transmission scheme, proposed by [1] among others, has recently attracted particular interest [2]. This scheme combines the well-known Orthogonal Frequency Division Multiplex (OFDM) and Code Division Multiple Access (CDMA) techniques. The OFDM modulation provides robustness against multipath as well as good spectral efficiency while the CDMA offers a highly flexible multiuser access by allocating to each user a distinct spreading sequence, which is orthogonal to the other users' sequences. Instead of spreading the binary information in the time domain as in the Direct Sequence CDMA technique, the MC-CDMA spreading is performed in the frequency domain in order to benefit from frequency diversity.

Thanks to OFDM, the inter-symbol interference (ISI) can be cancelled by adding a guard interval to each symbol. The mobile radio channel can be represented by a single fading coefficient per subcarrier, which has to be estimated for

coherent detection. For this purpose, we may apply similar channel estimators as those already used in pure OFDM systems, *e.g.*, DVB-T [3][4]. This paper considers channel estimation based on the insertion of pilot symbols as it has been investigated in [5] and [6]. Performance analysis, however, has only been given for the case of full system load, *i.e.*, for a maximum number of active users. Regarding the operator's point of view, MC-CDMA transmission systems have also to be optimized for lower system loads.

This paper addresses the influence of pilot symbol-based channel estimation on the MC-CDMA system performance in the case of a variable number of active users. We first present the structure of our MC-CDMA mobile radio transmission system in section II and recall some basic analytical expressions for the forward link and Maximum Ratio Combining (MRC) detection. In section III, the impact of erroneous channel estimation on the decision variable is analyzed, which emphasizes the influence of the pilot symbol power. We propose to adapt the pilot symbol power with respect to the system load so as to keep the SNR loss due to channel estimation errors constant. The adaptation rule, developed in section III for MRC, is generalized to other common detection techniques, such as minimum mean square error combining (MMSEC) or parallel interference cancellation (PIC), and validated by computer simulations in section IV. We also provide simulation results of the overall SNR per user required for a certain BER, when the rule is applied to typical indoor and outdoor channel configurations. We finally give concluding remarks in section V.

II. SYSTEM DESCRIPTION

The receiver structure of an MC-CDMA transmission system for the forward link is shown in Fig. 1. We assume that K independent bit streams corresponding to different users are multiplexed on the same subcarrier set chosen out of N_c subcarriers. The data bearing symbol before frequency mapping can be expressed as

$$x(i, l) = \sum_{k=0}^{K-1} d^k(i) c^k(l) \quad (1)$$

Here, i is the time index and will be dropped in the sequel for brevity. l is the subcarrier index, $l = 0 \dots L-1$, where L is the spreading code length. d^k is the data symbol (e.g., QPSK) of the k -th user and $c^k(l)$ is the element of the spreading code of the k -th user corresponding to the l -th subcarrier. The spreading codes have to be chosen in such a way that they provide orthogonality among users. Without loss of generality, we use Walsh-Hadamard codes as an ideally orthogonal code set. The length of the spreading code, L , can be chosen according to the frequency diversity offered by the channel. Thus, when applying an appropriate frequency mapping, we can consider that the fading on the subcarriers, on which the data symbol is spread, is uncorrelated. This is assumed for the following analytical study. However, as the channel properties change rapidly in mobile systems, we also may have correlated fading coefficients. This is considered in section IV.

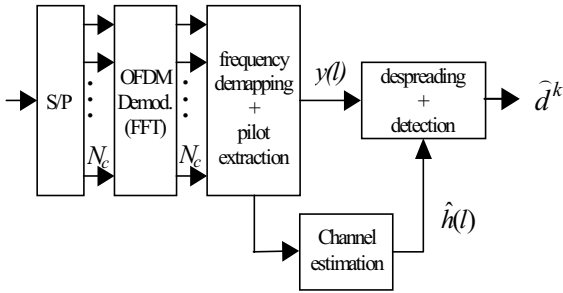


Fig 1: MC-CDMA receiver structure

The data bearing symbols are transmitted on narrowband subcarriers, each having a bandwidth of $\Delta f = 1/T_s$, where T_s is the OFDM symbol duration. After OFDM modulation, a guard interval, Δ , in form of a cyclic prefix is added to the symbol. The length of the guard interval has to be superior to the delay-spread for ISI-free transmission. The multipath channel can be represented by a single complex fading coefficient per subcarrier. The received symbol on each subcarrier after OFDM-demodulation can, therefore, be written as

$$y(l) = \sum_{k=0}^{K-1} h(l) d^k c^k(l) + n(l) \quad (2)$$

$h(l)$ and $n(l)$ are the Gaussian distributed complex channel coefficient and the AWGN sample of variance σ_N^2 on the l -th subcarrier, respectively. Among the multitude of single- and multiuser algorithms applicable for data detection, see e.g., [7] [8], we focus on MRC single user detection in order to derive intelligible analytical results. MRC applies the combining gain $q^g(l) = c^g(l) h^*(l)/L$ to the received signal on each subcarrier. The decision variable of

the desired user g , in the case of perfect channel knowledge, is

$$\hat{d}^g = d^g \underbrace{\frac{1}{L} \sum_{l=0}^{L-1} |h(l)|^2}_{\text{desired signal}} + \underbrace{\sum_{k=0, k \neq g}^{K-1} d^k \frac{1}{L} \sum_{l=0}^{L-1} |h(l)|^2 c^k(l) c^g(l)}_{\text{MAI}} + \underbrace{\frac{1}{L} \sum_{l=0}^{L-1} n(l) c^g(l) h^*(l)}_{\text{noise}} \quad (3)$$

Expression (3) is composed of three parts: The desired symbol multiplied by a coefficient depending on the fading, the multiple access interference (MAI), arising from the loss of orthogonality among users caused by the fading channel, and the residual noise after despreading. We will assume in the sequel that the channel is power normalized, i.e., $E[h(l)h^*(l)] = 1$ and that the energy of the data symbol d^k is $E_s = E[d^k d^{k*}]$ for each user. In this case, the noise variance after despreading is reduced by a factor L compared to σ_N^2 . The MAI can be considered as a Gaussian random variable for a large number of users. For MRC and independent fading coefficients the total error variance can be approximated by

$$\sigma_{MRC}^2 = \frac{(K-1)E_s}{L} + \frac{\sigma_N^2}{L} \quad (4)$$

These results are valuable for perfect channel estimation. In the next section, we investigate the impact of channel estimation errors.

III. IMPACT OF CHANNEL ESTIMATION

Channel estimation for MC-CDMA can be performed by inserting pilot symbols in the MC-CDMA frame according to a predefined pilot symbol grid [5] [6] similar to those already used in pure OFDM systems. The fading coefficients of subcarriers required for coherent detection are then observed on pilot positions. From these observations, the channel fading coefficients of all frame positions are estimated by interpolation.

A. Spectral efficiency and overall SNR

Introducing pilot symbols creates an overhead, which reduces the transmission spectral efficiency. If we consider an MC-CDMA frame consisting of N_c subcarriers and N_f consecutive MC-CDMA symbols, the overhead O_p is defined as the ratio between the number of pilot positions N_{pilots} and the total number of positions in the MC-CDMA frame, i.e.,

$$O_p = \frac{N_{pilots}}{N_c N_f} \quad (5)$$

The overall SNR per user taking into account the overhead is then defined as

$$SNR = 10 \log \left(\frac{KE_s(1-O_p) + O_p E_p}{\sigma_N^2 \cdot K(1-O_p)} \right) \quad (6)$$

Here, E_p is the power allocated to a pilot symbol. E_s is the power of the data symbol of one user on one subcarrier, what leads to an average power of the data bearing symbol, $x(l)$, per subcarrier of KE_s . From this expression, it is obvious that if we enhance the reliability of the channel estimates by increasing the pilot symbol power, the required SNR on the data symbols decreases while the part of the overall SNR spent for pilots increases according to the overhead. Thus, the main challenge of channel estimation is to determine O_p and E_p in order to minimize the overall SNR required by the transmission for achieving a given performance level, *e.g.*, BER of 10^{-2} after symbol detection. In systems with high overhead, the application of boosted pilots may be disadvantageous, while, in systems with low overhead, boosted pilots may help to keep the data SNR on a reasonable level, thus minimizing the overall SNR.

B. Channel estimation errors

In general, we may model the impact of erroneous channel estimation as an estimation error η corrupting the channel state information \hat{h} on each frame position. Thus, we get the following estimate on each subcarrier:

$$\hat{h}(l) = h(l) + \eta(l) \quad (7)$$

where the error $\eta(l)$ can be considered as Gaussian noise and independent of the channel coefficient. The latter assumption is particularly true if Wiener-filters are used for interpolation. The variance of the error depends on the pilot power, the subcarrier noise variance and the interpolation quality. It can be written as

$$\sigma_\eta^2 = \frac{\sigma_N^2}{\gamma E_p} \quad (8)$$

Here, γ takes into account an eventual smoothing gain, that can be achieved if different pilot symbols are affected by a correlated channel attenuation.

The channel estimation error according to (7) can be taken into account in the decision variable for MRC, by adding the following term to expression (3):

$$\begin{aligned} \Delta_{CE}^g &= \sum_{k=0}^{K-1} d^k \frac{1}{L} \sum_{l=0}^{L-1} h(l) \eta^*(l) c^k(l) c^g(l) \\ &\quad + \frac{1}{L} \sum_{l=0}^{L-1} n(l) c^g(l) \eta^*(l) \end{aligned} \quad (9)$$

Both parts can be considered as Gaussian random variables provided that we have a high spreading gain and a large number of users. The variance of the error on the decision variable uniquely caused by erroneous channel estimation can then be written as

$$\sigma_{CE}^2 = \frac{1}{L} \left(KE_s \sigma_\eta^2 + \sigma_N^2 \sigma_\eta^2 \right) \quad (10)$$

Expression (10) contains two terms: The cross-product of the two noise terms and a term depending on the system load, which can be seen as some kind of additional MAI that increases linearly with the number of active users.

Hence, the total error variance in the case of channel estimation errors and MRC detection can be expressed using (4) and (10) as

$$\tilde{\sigma}_{MRC}^2 = \frac{1}{L} \left((\sigma_\eta^2 + 1) KE_s + (\sigma_\eta^2 + 1) \sigma_N^2 - E_s \right) \quad (11)$$

If $\bar{\sigma}_N^2$ defines the noise variance tolerated at a certain BER for perfect channel knowledge ($\sigma_\eta^2 = 0$), a lower noise variance $\tilde{\sigma}_N^2$ will be required to achieve the same BER with erroneous channel estimation ($\sigma_\eta^2 > 0$). Hence, we may express the SNR loss due to channel estimation errors by a factor λ defined as

$$\lambda = \frac{\bar{\sigma}_N^2}{\tilde{\sigma}_N^2}; \quad \lambda > 1 \quad (12)$$

C. Pilot power optimization

Looking at expression (8), that emphasizes the influence of the pilot symbol power on σ_η^2 and, therefore, on σ_{CE}^2 , and considering expression (6), it gets obvious that there is a trade-off between the power allocated to pilot symbols and the performance degradation resulting from channel estimation errors.

A frequent choice is to set the pilot symbol power equal to the average power of the data symbols on each subcarrier [5], *i.e.*, $E_p = KE_s$. In this case, the quality of the channel observations decreases rapidly for low system loads, what leads to excessive channel estimation errors, which cannot be compensated by a tolerable increase of the data SNR.

In contrast, we propose an approach that takes into account the variable load property of MC-CDMA transmission systems. We choose an operation point, *e.g.*, a BER of 10^{-2} after symbol detection, and a tolerable SNR loss, λ . The operation point should then be attained with the constant loss λ for all system loads. For instance, we may consider $\lambda = 1$ dB as a realistic value required for the estimation of mobile radio channels.

In order to ensure a constant BER performance, the total error variances for perfect channel estimation and for erroneous channel estimation have to be kept on the same level. This means

$$\sigma_{MRC}^2 = \tilde{\sigma}_{MRC}^2 \quad (13)$$

For satisfying this condition at a given operation point, we have to adapt the pilot symbol power as the system load varies. If we insert (8), (11) and (12) in (13) we get an adaptation rule for the pilot symbol power taking the following form:

$$E_p = \frac{1}{\gamma} \left(\frac{1}{\lambda} KE_s + \frac{L \sigma_{MRC}^2 + E_s}{\lambda^2 - \lambda} \right) \quad (14)$$

The analytical result (14) shows that the criterion of having a constant SNR loss due to channel estimation errors at a certain BER level independently of system load leads to an adaptation rule of the pilot symbol power, which is linear with the number of active users. It was developed here for MRC detection and uncorrelated fading on the subcarriers and will be extended to other detection techniques and channel configurations in the following section.

IV. EXTENSION AND SIMULATION RESULTS

In this section, we validate the pilot power optimization as proposed in (14) by simulation results. Furthermore, as the performance of the MRC detector degrades rapidly with increasing system load, we shall consider more sophisticated detection techniques and verify by simulation results how the adaptation rule applies to them. In the following, we use MMSEC detection, which has been shown to be a very efficient single user detection technique [8], and a PIC scheme based on MMSEC detection [9].

Fig. 2 shows the pilot power required to ensure a constant SNR loss $\lambda = 2$ dB at a BER equal to $2 \cdot 10^{-2}$ as a function of system load for MRC, MMSEC and PIC with uncorrelated fading. The spreading gain was set to $L = 64$. No smoothing gain is considered, *i.e.*, $\gamma = 1$. Analytical and simulation results for MRC match quite well, what confirms the validity of the approximations used to derive equation (14). Furthermore, the simulation results show that the linearity of the adaptation rule is conserved for MMSEC and PIC. Hence, we may formulate a more general adaptation rule for the pilot symbol power in the following way:

$$E_p = \alpha(K-1) + \beta \quad (15)$$

β designates an offset, which is the pilot power required at the operation point for a single user. It clearly depends on the BER level, the tolerated data SNR loss λ and the spreading gain (cf. (14)). From Fig. 2, we see that β can be considered independent of the detection technique. α designates the slope, *i.e.*, the increase in pilot power required for each additional user. From (14), we conclude that this parameter only depends on the data SNR loss λ , in the case of MRC. For other detection techniques, however, α changes. We note again that, up to now, we only considered uncorrelated fading on the subcarrier set on which the data symbol is transmitted. Further simulations showed that equation (15) still holds in the case of correlation among the subcarriers, with an influence on the parameter α . As expected from equation (14), a smoothing gain of the channel estimator reduces the parameters of (15) by the corresponding factor γ .

In a general way, the parameters of the adaptation rule (15) may be determined by analysis, as it was shown for MRC

and uncorrelated fading in section III, or, in the case where analysis gets difficult, by simulation of a few points of the straight line given by (15).

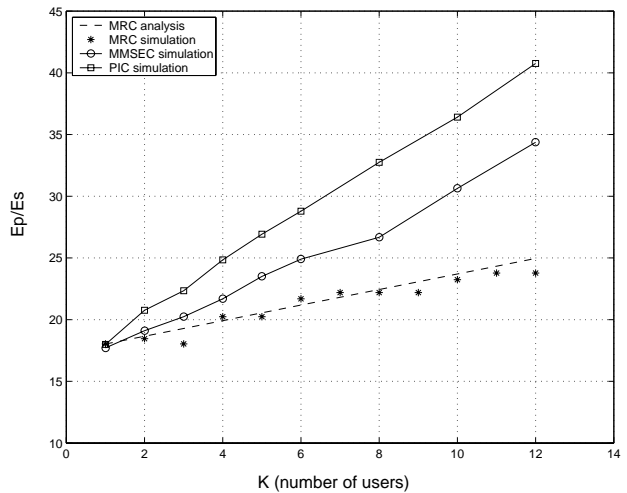


Fig 2: Required pilot power versus system load

$\lambda = 2$ dB, uncorrelated fading, BER $2 \cdot 10^{-2}$

In order to evaluate the performance of the pilot power adaptation method, we now consider the resulting overall SNR per user taking into account the overhead according to (6). We present simulation results for indoor (cf. Fig. 3) and outdoor (cf. Fig. 4) MC-CDMA transmission systems with MMSEC single user detection at an uncoded BER level of 10^{-3} . For a given overhead, we compare the minimized overall SNR obtained with the adaptation rule (15) to setting the pilot power to the average data symbol power (*i.e.*, $E_p = KE_s$).

Fig. 3 presents results obtained with an indoor system as considered in the European BRAN Hiperlan/2 standardization project [10] (*i.e.*, carrier frequency: 5 GHz, bandwidth: 20 MHz, maximum delay spread: 800 ns, maximum Doppler-shift: 50 Hz). We choose a spreading factor equal to the number of subcarriers, *i.e.*, $L = 64$. Note that there is a certain fading correlation on adjacent subcarriers. This allows an overhead of 1%, which is sufficient to provide a reliable estimate of the channel coefficients with a smoothing gain $\gamma = 6$ dB.

The results in Fig. 3 illustrate again the trade-off between the SNR loss tolerated with channel estimation, λ , and the SNR loss resulting from the pilot overhead. Thanks to the adaptation rule (15), we can easily determine the tolerable SNR loss leading to an optimized overall SNR. Here, we should choose an SNR loss of $\lambda = 0.5$ dB, because it minimizes the overall SNR for all system loads. We also see that this choice clearly outperforms setting $E_p = KE_s$, especially for low system loads. There is a gain of about 0.5 dB for full load and more than 1 dB at low loads.

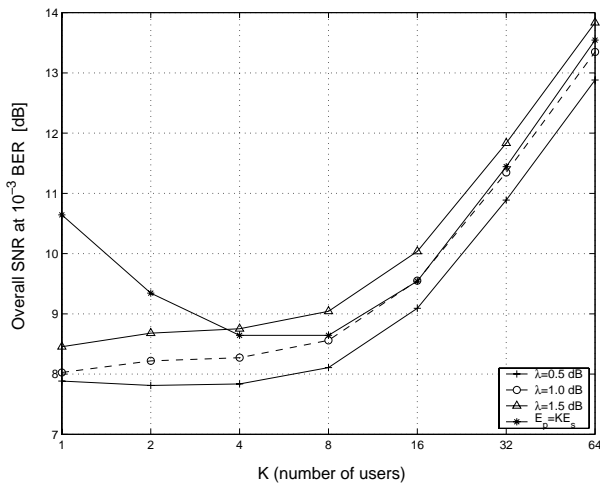


Fig 3: Overall SNR versus system load

Indoor channel, BER 10^{-3} , $L = 64$, $O_p = 1\%$, $\gamma = 6$ dB

Fig. 4 presents results obtained with an outdoor channel. We assume a maximum delay spread of $5 \mu\text{s}$ and a maximum Doppler shift of 500 Hz (100km/h). Bandwidth and carrier frequency stay unchanged. The number of subcarriers is 512 and the spreading factor is 128. We assume that we have uncorrelated fading on the subcarriers thanks to frequency interleaving. The required overhead is 5% and we benefit from a 4 dB smoothing gain.

For systems with high spreading gains, like the one considered in Fig. 4, the need of pilot power adaptation is even more apparent. Thanks to the adaptation rule, we can minimize the overall SNR for all loads by choosing a loss $\lambda = 1$ dB. Even with an overhead higher than in the indoor system, this yields an advantage of almost 0.5 dB at full load and up to 1.5 dB for low system loads, compared to $E_p = KE_s$.

Note that the SNR loss λ may also be selected adaptively according to system load in order to minimize the overall SNR.

V. CONCLUSION

We investigated an MC-CDMA system with pilot symbol based channel estimation in the realistic non-full load case, *i.e.*, when the number of spreading sequences allocated to the active users is lower than the total number of available spreading sequences. By analysis of the impact of channel estimation errors on detection, we developed a simple rule to adapt the pilot symbol power to the number of active users in the system. The rule is based on the criterion of a constant SNR loss on the data symbols due to erroneous channel estimation, independently of system load. Thanks to its simplicity the rule can easily be applied to various system configurations by measuring or calculating

the two required parameters. We further showed that this rule enables us to minimize the required overall SNR and that it is particularly beneficial for systems with high spreading gain and low overhead.

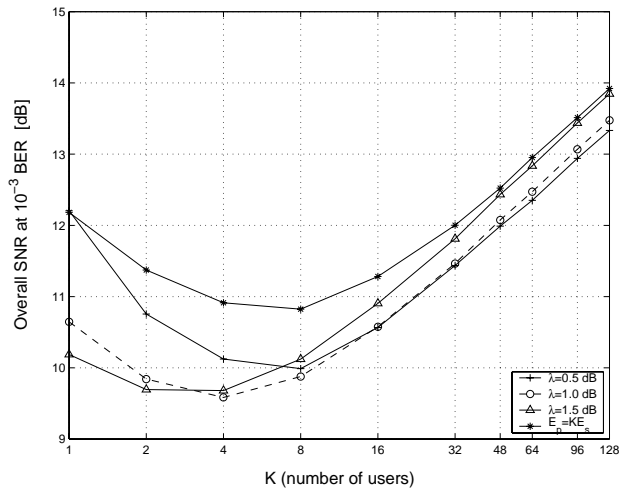


Fig 4: Overall SNR versus system load

Outdoor channel, BER 10^{-3} , $L = 128$, $O_p = 5\%$, $\gamma = 4$ dB

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