Optimum and Sub-Optimum Multiuser Detection Based on Sphere Decoding for Multi-Carrier Code Division Multiple Access Systems

Loïc Brunel
Mitsubishi Electric ITE – Telecommunication Laboratory
80, avenue des Buttes de Coësmes – Immeuble Germanium – 35700 Rennes – France

Abstract – **When performed using an exhaustive search, the Maximum Likelihood (ML) joint detection of all users in a Multi-Carrier Code Division Multiple Access (MC-CDMA) system has a prohibitive complexity, growing exponentially with the number of users and the number of bits in each modulation symbol. An ML multiuser detection algorithm has recently been proposed, with a complexity growing polynomially with the number of users, independent of the modulation size. The MC-CDMA system is modelled as a sphere packing lattice and a low-complexity optimum lattice decoder, called the sphere decoder, is applied to jointly detect all users. We propose sub-optimum simplifications, based on orthogonal projection of the received signal on a facet of the lattice constellation, to further decrease the complexity. Simulation results are shown with up to 64 users transmitting 16-QAM symbols.**

I. Introduction

The Multi-Carrier Code Division Multiple Access (MC-CDMA) technique, initially proposed by [1][2][3], efficiently combines the Orthogonal Frequency Division Multiplex (OFDM) modulation and the code division multiple access technique. Each user symbol is transmitted over several sub-carriers. On each sub-carrier, this signal is multiplied by a distinct element of a user-specific signature. In other words, the transmitted symbol is spread before OFDM modulation. Low cross-correlation values between signatures are desirable to enable user separation at the receiver. Thanks to the guard interval between OFDM symbols, MC-CDMA systems do not suffer from inter-symbol interference (ISI) and quasi-synchronism between users may be obtained in the uplink. Furthermore, the OFDM modulation takes advantage of a large frequency diversity. To combat the multiuser interference due to the loss of orthogonality introduced by the transmission on a multipath channel, various single-user and multiuser detection techniques have been proposed [4]. Among them, the optimum multiuser detection, based on a Maximum Likelihood (ML) exhaustive search [2], has a prohibitive complexity, growing exponentially with the number of users and the number of bits per modulation symbol.

A new ML detection algorithm with low complexity has recently been proposed [5]. This algorithm, called the Sphere Decoding algorithm, originally employed for sphere packing lattice decoding [6][7], has also been proposed to simplify the ML multiuser detection in Direct-Sequence Code Division Multiple Access (DS-CDMA) [8]. The MC-CDMA receiver models the Maximum Ratio Combining (MRC) output as a multi-dimensional sphere packing lattice point corrupted by additive noise. The lattice sphere decoder is then applied to jointly detect all users. Its complexity is a polynomial function of the number of users and is independent of the modulation size. Thus, it allows optimum performance even for full-loaded systems using large modulations. The absence of ISI and the synchronism assumption make the MC-CDMA a particularly suitable system for lattice representation and sphere decoding. However, for a large number of users, the decoding complexity may still be too high for very noisy received signals. In this case, we propose sub-optimum simplifications by orthogonal projection, which speed up the decoding process with low performance loss.

The paper is organised as follows: Section II describes the synchronous MC-CDMA system and the corresponding lattice representation. In section III, the sphere decoder is explained for lattice constellations and then applied to MC-CDMA. Simplifications are described in section IV. Simulation results for downlink are presented in section V and compared to those of classical sub-optimum detection algorithms before conclusion.

II. Lattice representation of an MC-CDMA system

Let us represent a synchronous MC-CDMA system using a sphere packing lattice [9]. A κ-dimensional sphere packing lattice of $\mathbf{R}^\nu$ is a discrete subgroup (or a $\mathbf{Z}$-module) with rank κ of $\mathbf{R}^\nu$. We denote $\mathbf{R}$ the real space and $\mathbf{Z}$ the integer ring. Each point $\mathbf{x}$ of lattice Λ may be written as the linear combination of κ basis row vectors $\mathbf{v}_k$:

$$\mathbf{x} = b_1\mathbf{v}_1 + \ldots + b_\kappa\mathbf{v}_\kappa \text{ where } b_k \in \mathbf{Z},\ \forall k = 1,\ldots,\kappa \qquad (1)$$

Vectors $\mathbf{v}_k$ compose the $\kappa \times \nu$ lattice *generator matrix* $\mathbf{G}$. Thus, $\mathbf{x} = \mathbf{bG}$ where $\mathbf{b} = (b_1,\ldots,b_\kappa) \in \mathbf{Z}^\kappa$.

We consider a synchronous MC-CDMA system with K users as depicted on Fig. 1. At time i and for user k, the transmitted symbol $b_k(i)$ is spread by a Walsh-Hadamard signature $\mathbf{c}_k = (c_{k1},\ldots,c_{kL})$, with length L, orthogonal to other users' signatures. The L obtained chips are transmitted with amplitude ω_k on the L different sub-carriers building an OFDM symbol. Each symbol $b_k(i)$ is taken from $A_k = \{b_{\min,k}, b_{\min,k}+1,\ldots, b_{\max,k}\}^2$, a complex modulation alphabet with cardinality $|A_k|$, *i.e.*, the real and imaginary parts $b_k^R(i)$ and $b_k^I(i)$ belong to an integer alphabet, a Pulse Amplitude Modulation (PAM). *E.g.*, $b_{\min,k} = 0$ and $b_{\max,k} = 3$ for a 16-QAM (Quadrature Amplitude Modulation) alphabet. This constellation is obviously not optimal from the energy point of view, but a 16-QAM constellation with minimum energy can also be used at the price of a few additional operations [5]. A guard interval Δ is inserted to absorb ISI. We denote $s_k(i)$ the modulated signal filtered by a frequency selective multipath channel. After addition of interfering users' signals, $\sum_{k'\neq k} s_{k'}(i)$, and Additive White Gaussian Noise (AWGN), OFDM demodulation is performed. The channel is assumed non frequency selective on the sub-carrier bandwidth and is thus described by a single complex coefficient $h_{k\ell}(i)$ for each user k and each sub-carrier ℓ. We denote $\mathbf{C}(i) = [C_{k\ell}(i)]$ the $K \times L$ matrix merging spreading and channel coefficients for all users: $C_{k\ell}(i) = c_{k\ell} h_{k\ell}(i)$. At time i, $\mathbf{r}(i) = (r_1(i),\ldots,r_L(i))$, the received vector, may be expressed as

$$\mathbf{r}(i) = \mathbf{b}(i)\mathbf{D}_{\omega}\mathbf{C}(i) + \boldsymbol{\eta}(i) \qquad (2)$$

where vector $\mathbf{b}(i) = (b_1(i),\ldots,b_K(i))$ contains the K transmitted symbols, $\mathbf{D}_{\omega} = \mathrm{diag}(\omega_1,\ldots,\omega_K)$ contains the user amplitudes and $\boldsymbol{\eta}(i) = (\eta_1(i),\ldots,\eta_L(i))$ is the AWGN vector.

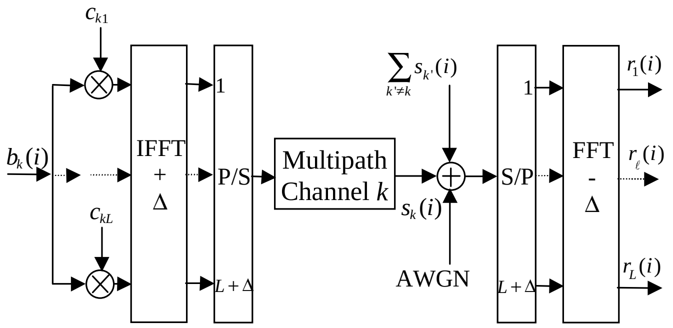

Fig. 1: MC-CDMA transmitter and OFDM receiver.

We now focus on the ML multiuser detection using the received signal given by (2). It can be shown [5] that a sufficient statistic for the ML detection of transmitted vector $\mathbf{b}(i)$ is the observation vector $\mathbf{y}(i) = (y_1(i),\ldots,y_K(i))$, where $y_k(i)$ is the MRC output for user k [3]:

$$y_k(i) \stackrel{\Delta}{=} \sum_{\ell=1}^{L} c_{k\ell}^{*}(i) h_{k\ell}^{*}(i) r_\ell(i) \qquad (3)$$

$\mathbf{y}(i)$ may be written in a matrix form from (3):

$$\mathbf{y}(i) \stackrel{\Delta}{=} \mathbf{r}(i)\mathbf{C}^H(i) \qquad (4)$$

where $\bullet^H$ denotes the transpose-conjugate. By including (2) in (4), we obtain $\mathbf{y}(i)$ as a function of $\mathbf{b}(i)$:

$$\mathbf{y}(i) = \mathbf{b}(i)\mathbf{D}_{\omega}\mathbf{C}(i)\mathbf{C}^H(i) + \mathbf{n}(i) \qquad (5)$$

where noise $\mathbf{n}(i) = (n_1(i),\ldots,n_K(i)) = \boldsymbol{\eta}(i)\mathbf{C}^H(i)$.

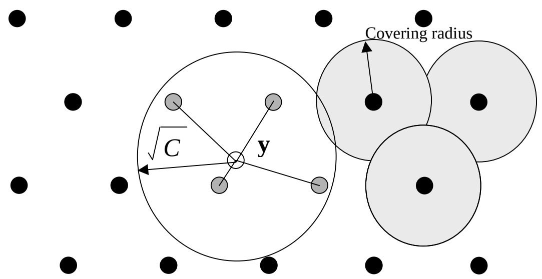

Fig. 2: Geometrical representation of sphere decoding ($\kappa = 2$).

We now write all previously defined complex vectors (resp. matrices) of size K (resp. $K \times K$) as real vectors (resp. matrices) of size $2K$ (resp. $2K \times 2K$):

$$e.g.,\ \mathbf{b}_2(i) = \left(b_1^R(i), b_1^I(i), \ldots, b_K^R(i), b_K^I(i)\right)$$

$$\mathbf{D}_{\omega,2} = \mathrm{diag}\left(\omega_1, \omega_1, \ldots, \omega_K, \omega_K\right) \qquad (6)$$

$$\mathbf{R}_2(i) = \begin{bmatrix} R_{11}^R & R_{11}^I & \cdots & R_{1K}^R & R_{1K}^I \\ -R_{11}^I & R_{11}^R & \cdots & -R_{1K}^I & R_{1K}^R \\ \vdots & \vdots & & \vdots & \vdots \\ R_{K1}^R & R_{K1}^I & \cdots & R_{KK}^R & R_{KK}^I \\ -R_{K1}^I & R_{K1}^R & \cdots & -R_{KK}^I & R_{KK}^R \end{bmatrix}$$

where $\mathbf{R}(i) = [R_{ij}] = \mathbf{C}(i)\mathbf{C}^H(i)$. Each complex value a is expressed as $a = a^R + j.a^I$. With the above notations, we obtain

$$\mathbf{y}_2(i) = \mathbf{b}_2(i)\mathbf{M}_2(i) + \mathbf{n}_2(i) \ \text{ with } \ \mathbf{b}_2(i) \in \mathbf{Z}^{2K} \qquad (7)$$

$\mathbf{y}_2(i)$ is a point of a lattice Λ_2 in $\mathbf{R}^{2K}$, with dimension $2K$ and generator matrix $\mathbf{M}_2(i) = \mathbf{D}_{\omega,2}\mathbf{R}_2(i)$, corrupted by an additive noise $\mathbf{n}_2(i)$ with covariance matrix $N_0\mathbf{R}_2(i)$. The multiple access system generates a point $\mathbf{b}_2(i)\mathbf{M}_2(i)$ belonging to a constellation *i.e.*, a finite subset of Λ_2, with size $|A_1| \times \ldots \times |A_K|$. Using the lattice representation allows us to apply the Sphere Decoding algorithm [6][7], a low complexity ML decoding algorithm.

III. Sphere decoding of a synchronous MC-CDMA system

Let us first describe the ML decoding on AWGN channel of a κ-dimensional lattice Λ in $\mathbf{R}^{\kappa}$ generated by a real $\kappa \times \kappa$ matrix $\mathbf{G}$. The decoder has to find the closest lattice point to the received vector *i.e.*, to minimise

$$m(\mathbf{y}/\mathbf{x}) = \sum_{k=1}^{\kappa} \left|y_k - x_k\right|^2 = \left\|\mathbf{y} - \mathbf{x}\right\|^2 \qquad (8)$$

where $\mathbf{y} = \mathbf{x} + \boldsymbol{\eta}$ is the received vector, $\boldsymbol{\eta} = (\eta_1,\ldots,\eta_\kappa)$ the Gaussian noise vector and $\mathbf{x} = (x_1,\ldots,x_\kappa)$ a point of Λ. $\boldsymbol{\eta}$ has real independent elements with zero mean and variance N_0. Lattice points $\{\mathbf{x} = \mathbf{bG}\}$ are obtained from data vectors $\mathbf{b} = (b_1,\ldots,b_\kappa) \in \mathbf{Z}^{\kappa}$. In practice, the set of data vectors is limited to a finite alphabet $A^{(\kappa)} \subset \mathbf{Z}^{\kappa}$ and an exhaustive ML decoder searches for the best point $\mathbf{x}$ among all points in the finite constellation. The sphere decoder restricts its computation to the points located inside a hypersphere with radius $\sqrt{C}$ centred on the received point as shown on Fig. 2 for $\kappa = 2$ and a constellation with 20 points. The following minimisation is performed to find the shortest vector $\mathbf{w}$ in translated set $\mathbf{y} - \Lambda$:

$$\min_{\mathbf{x}\in\Lambda} \left\|\mathbf{y} - \mathbf{x}\right\| = \min_{\mathbf{w}\in\mathbf{y}-\Lambda} \left\|\mathbf{w}\right\| \qquad (9)$$

The difference $\mathbf{w} = \boldsymbol{\xi}\mathbf{G}$, $\boldsymbol{\xi} = (\xi_1,\ldots,\xi_\kappa) \in \mathbf{R}^{\kappa}$, is a lattice point, whose coordinates ξ_i are expressed on the translated basis centred on received vector $\mathbf{y} = \boldsymbol{\rho}\mathbf{G}$, $\boldsymbol{\rho} = (\rho_1,\ldots,\rho_\kappa) \in \mathbf{R}^{\kappa}$. Since $\mathbf{w}$ must be located in a hypersphere with quadratic radius C centred on $\mathbf{y}$, we get:

$$\left\|\mathbf{w}\right\|^2 = \boldsymbol{\xi}\mathbf{G}\mathbf{G}^T\boldsymbol{\xi}^T \le C \qquad (10)$$

In the new coordinates' system defined by ξ, this hypersphere is changed into a hyperellipse centred at the origin. The Cholesky factorisation of the Gram matrix $\mathbf{\Gamma} = \mathbf{G}\mathbf{G}^T$ yields $\mathbf{\Gamma} = \mathbf{A}\mathbf{A}^T$, where lower triangular matrix $\mathbf{A}$ has elements a_{ij}. Using (9), it can be shown [6][7] that a point is inside the hyperellipse if and only if

$$\forall k = 1,\ldots, \kappa, \quad B_{\min,k} \le b_k \le B_{\max,k}$$

where (11)

$$B_{\min,k} = \left\lceil -\sqrt{\frac{1}{q_{kk}}\left(C - \sum_{\ell=k+1}^{\kappa} q_{\ell\ell}\left(\xi_\ell + \sum_{j=\ell+1}^{\kappa} q_{j\ell}\xi_j\right)^2\right)} + \rho_k + \sum_{j=k+1}^{\kappa} q_{jk}\xi_j \right\rceil$$

$$B_{\max,k} = \left\lfloor \sqrt{\frac{1}{q_{kk}}\left(C - \sum_{\ell=k+1}^{\kappa} q_{\ell\ell}\left(\xi_\ell + \sum_{j=\ell+1}^{\kappa} q_{j\ell}\xi_j\right)^2\right)} + \rho_k + \sum_{j=k+1}^{\kappa} q_{jk}\xi_j \right\rfloor$$

$q_{kk} = a_{kk}^{\ 2}$, $q_{kj} = a_{kj} / a_{jj}$, $\lceil x \rceil$ is the *ceil* function and $\lfloor x \rfloor$ the *floor* function. κ counters, one per dimension, enumerate all values of vector $\mathbf{b}$ such that lattice point $\mathbf{x} = \mathbf{b}\mathbf{G}$ is located within the quadratic distance C from the received point. Points located outside the considered hypersphere are never tested. Consequently, the complexity of this optimum decoding does not depend on the lattice constellation size $|\mathrm{A}^{(\kappa)}|$. Furthermore, the search is drastically speeded up by updating $\sqrt{C}$ with the last computed norm $\|\mathbf{w}\|$. Finally, the selected point $\mathbf{x}$ is the point associated to the minimum norm $\|\mathbf{w}\|$. Since the number of points located in the decoding hypersphere increases with radius $\sqrt{C}$, it must be carefully chosen. A large value slows down the algorithm, whereas the hypersphere may be empty if C is too small. By taking a search radius greater than the covering radius [9] depicted on Fig. 2, we ensure that the decoder will find at least a lattice point. However, this point does not necessarily belong to the finite constellation.

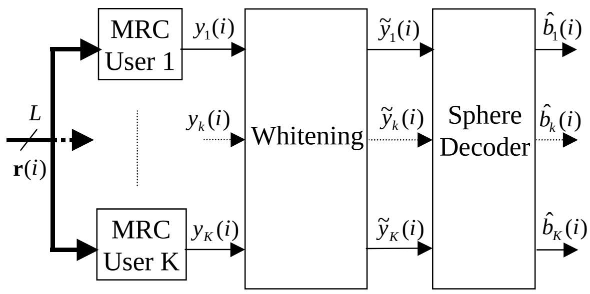

Fig. 3: MC-CDMA detector with sphere decoding (after FFT).

For multiuser detection, the sphere decoding is applied to the $2K$-dimensional lattice representing the MC-CDMA system, one time for each received point *i.e.*, for K users. Since the additive noise samples in (7) are correlated, we have to whiten the noise in the MRC outputs in order to match the above sphere decoding assumptions. To simplify notations, we omit all 'i' indices. The Cholesky factorisation of cross-correlation matrix $\mathbf{R}_2$ yields $\mathbf{R}_2 = \mathbf{W}_2\mathbf{W}_2^{\ T}$, where $\mathbf{W}_2$ is lower triangular. The whitened observation is

$$\tilde{\mathbf{y}}_2 = \mathbf{y}_2\mathbf{W}_2^{\ T^{-1}} = \mathbf{b}_2\mathbf{M}_2\mathbf{W}_2^{\ T^{-1}} + \tilde{\mathbf{n}}_2 \qquad (12)$$

where $E[\tilde{\mathbf{n}}_2^{\ T}\tilde{\mathbf{n}}_2] = N_0\ \mathbf{I}_{2K}$. This observation must be processed with the sphere decoder associated to a new lattice generated by matrix $\mathbf{G}_2 = \mathbf{D}_{\omega,2}\mathbf{W}_2$ in order to finally obtain the detected vector $\hat{\mathbf{b}}_2$ [5]. The receiver structure is depicted on Fig. 3. To ensure that $\hat{\mathbf{b}}_2$ belongs to the transmitted constellation, bounds in (11) must be appropriately restricted:

$$\forall k = 1,\ldots, 2K, \quad B'_{\min,k} \le b_{2,k} \le B'_{\max,k}$$

where
$$B'_{\min,k} = \max(B_{\min,k}, b_{\min,\lceil k/2 \rceil}) \qquad (13)$$
$$B'_{\max,k} = \min(B_{\max,k}, b_{\max,\lceil k/2 \rceil})$$

These restricted bounds avoid considering lattice points located in the search sphere but not belonging to the constellation. This restriction, preserving the algorithm optimality, yields a complexity reduction, which is higher with smaller constellations. Thus, the obtained complexity is not independent of the constellation size anymore.

IV. Simplifications by Orthogonal Projection

Very noisy received vectors are received far away from the constellation. Hence, to preserve optimality, we must choose a search radius much larger than the covering radius and the algorithm becomes very slow. To ensure realistic decoding speed for noisy symbols, we propose to orthogonaly project them on the lattice constellation. The sub-optimality introduced by projection is expected to have a small impact on performance as it acts on very noisy and thus unreliable symbols.

A. Selection of the projection sub-space

Since the number of points per dimension corresponds to the number of modulation symbols in phase or quadrature, this number is constant for a given dimension, whatever the values of other coordinates *i.e.*, whatever the symbols transmitted by other users. Thus, the integer coordinates define a hypercube and the lattice constellation is a parallelogram, as depicted on Fig. 4a and 4b respectively. Fig. 4 also shows how the dimension of the affine projection sub-space depends on the received vector location. It is easier to transpose the problem from the lattice real space (Fig. 4b) into the integer coordinate space (Fig. 4a). Instead of projecting directly on the constellation facet, we prefer assigning the following extended bounds to each dimension k: $f_{\min,k} = b_{\min,k} - \alpha_k$ and $f_{\max,k} = b_{\max,k} + \alpha_k$ where α_k is a real positive value.

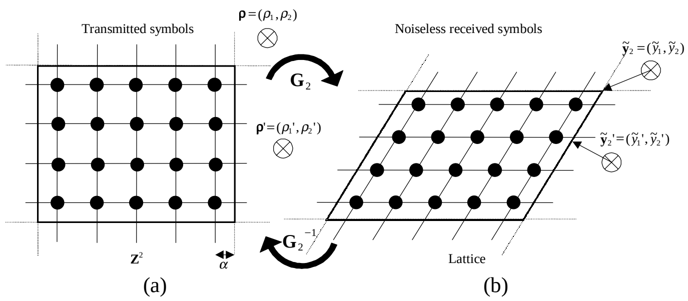

Fig. 4: Determination of the projection characteristics ($2K = 2$).

As depicted on Fig. 4, we use vector $\boldsymbol{\rho} = \tilde{\mathbf{y}}_2\mathbf{G}_2^{-1}$ to determine on which affine sub-space we project the received point. Indeed, this vector shows us the received point's position with respect to the constellation and the type of projection we have to carry out: projection on a point for $\tilde{\mathbf{y}}_2$ or projection on a straight line for $\tilde{\mathbf{y}}_2$'. We choose the following sub-optimum choice criterion: If Q dimensions are such that $\rho_k \leq f_{\min,k}$ or $\rho_k \geq f_{\max,k}$, we assign $f_{\min,k}$ or $f_{\max,k}$ to these values and we project on the corresponding affine sub-space A_N with dimension $N = 2K - Q$.

B. Definition of the projection on a constellation facet

Without loss of generality, we assume $\rho_k \geq f_{\max,k}$, $k = 1,\ldots,Q$. Let $\mathbf{y}^{\mathrm{P}} = \boldsymbol{\rho}^{\mathrm{P}}\mathbf{G}_2$ be the orthogonal projection on the affine sub-space A_N with dimension N of the whitened vector $\tilde{\mathbf{y}}_2$. According to the projection criterion, the Q first coordinates are fixed to $f_{\max,k}$: $\rho_k^{\mathrm{P}} = f_{\max,k}$ for $k = 1,\ldots,Q$. Thus,

$$\mathbf{y}^{\mathrm{P}} = \boldsymbol{\rho}^0\mathbf{G}^0 + \boldsymbol{\rho}^1.\mathbf{G}^1 = \mathbf{y}^0 + \boldsymbol{\rho}^1.\mathbf{G}^1 \tag{14}$$

where $\boldsymbol{\rho}^0 = (f_{\max,1},\ldots, f_{\max,Q})$ is a $1 \times Q$ row vector, $\mathbf{y}^0$ a $1 \times 2K$ row vector, $\boldsymbol{\rho}^1$ a $1 \times N$ row vector representing the degrees of freedom, $\mathbf{G}^0$ a $Q \times 2K$ matrix including the Q first rows of $\mathbf{G}_2$ and $\mathbf{G}^1$ a $N \times 2K$ matrix, including the N last rows of $\mathbf{G}_2$ and building a basis of the vector sub-space V_N. The projection on this vector sub-space, generated by rows of $\mathbf{G}^1$, can be easily written by

$$\mathbf{y}^{\mathrm{P}} = \left(\tilde{\mathbf{y}}_2 - \mathbf{y}^0\right)\mathbf{G}^{1^+}.\mathbf{G}^1 + \mathbf{y}^0, \tag{15}$$

where $\mathbf{G}^{1^+} = \mathbf{G}^{1^{\mathrm{H}}}\left(\mathbf{G}^1.\mathbf{G}^{1^{\mathrm{H}}}\right)^{-1}$ is the pseudo-inverse of $\mathbf{G}^1$.

The projected point is closer to the constellation, hence the sphere detection in the $2K$-dimensional space will be faster. The gain in complexity will be all the higher as the signal to noise ratio will be small.

C. Sphere decoding in the reduced dimension lattice

Instead of considering the $2K$-dimensional lattice for sphere decoding, we may limit us to the N-dimensional lattice included in the projection sub-space A_N. Indeed, points of lattice Λ_2 in $\mathbf{R}^{2K}$ contained in this N-dimensional affine sub-space are also points of a lattice Λ' in $\mathbf{R}^N$. If sphere decoding is performed on lattice Λ' with dimension $N \leq 2K$, the detection is further speeded up at the cost of a slight approximation: the detection simplification is based on the assumption that the transmitted point belongs to the facet on which the whitened vector $\tilde{\mathbf{y}}_2$ has been projected, although the closest lattice point of Λ_2 might not belong to this facet.

To proceed to the decoding in lattice Λ', the received point must be directly projected on the constellation facet: If Q dimensions are such that $\rho_k \leq f_{\min,k}$ or $\rho_k \geq f_{\max,k}$, the correspon-ding coordinates are fixed to $b_{\min,k}$ or $b_{\max,k}$. Without loss of generality, we assume $\rho_k \geq f_{\max,k}$, $k = 1,\ldots,Q$. According to the projection criterion, the Q first coordinates are fixed to $b_{\max,k}$: $\rho_k^{\mathrm{P}} = b_{\max,k}$ for $k = 1,\ldots,Q$. From (12), we then write the projection as a point of Λ_2 corrupted with additive noise:

$$\mathbf{y}^{\mathrm{P}} = \mathbf{b}^{\mathrm{P}}\mathbf{G}_2 + \mathbf{n}^{\mathrm{P}} = \mathbf{b}^0\mathbf{G}^0 + \mathbf{b}^1\mathbf{G}^1 + \mathbf{n}^{\mathrm{P}} \tag{16}$$

where $\mathbf{b}^0 \in \mathbf{Z}^Q$, $\mathbf{b}^1 \in \mathbf{Z}^N$, $\mathbf{n}^{\mathrm{P}}$ is the noise vector.

Since the transmitted point is assumed to belong to the projection facet, the Q first coordinates of the detected vector will be $\mathbf{b}^0 = (b_{\max,1},\ldots,b_{\max,Q})$. Let us now focus on the detection of vector $\mathbf{y}^1$ with dimension $2K$ belonging to the vector sub-space V_N and obtained by subtracting the constant vector $\mathbf{y}^0 = \boldsymbol{\rho}^0\mathbf{G}^0 = \mathbf{b}^0\mathbf{G}^0$ from (16):

$$\mathbf{y}^1 = \mathbf{y}^{\mathrm{P}} - \mathbf{y}^0 = \mathbf{b}^1\mathbf{G}^1 + \mathbf{n}^{\mathrm{P}} \tag{17}$$

Matrix $\mathbf{G}^1$ contains N basis vectors with size $2K$. Thus, $\mathbf{y}^1$ is noisy point of a N-dimensional lattice in $\mathbf{R}^{2K}$. In order to be treated by the decoding as presented in section III, the observation vector must be a noisy point of an N-dimensional lattice Λ' in $\mathbf{R}^N$. We have to find a $N \times N$ matrix $\mathbf{B}^1$ generating a lattice equivalent to the lattice generated by $\mathbf{G}^1$ in $\mathbf{R}^{2K}$. Equation (10) is the single relation employed in the sphere decoding and involving the lattice structure, through the Gram matrix of the generator matrix. In order that the lattice generated by $\mathbf{B}^1$ be equivalent to the lattice generated by $\mathbf{G}^1$, both matrices must have the same Gram matrix. Thus, $\mathbf{B}^1$ can be found by Cholesky factorisation of the Gram matrix of $\mathbf{G}^1$:

$$\mathbf{G}^1\mathbf{G}^{1^T} = \mathbf{B}^1\mathbf{B}^{1^T} \tag{16}$$

Let $\mathbf{U}$ be the $2K \times N$ transfer matrix such that $\mathbf{B}^1 = \mathbf{G}^1\mathbf{U}$. It can be shown that

$$\mathbf{U} = \mathbf{G}^{1^T}\left(\mathbf{B}^{1^{-1}}\right)^T \tag{17}$$

If the row vector $\mathbf{x}^1 = \mathbf{b}^1\mathbf{G}^1$ with size $2K$ is a point of Λ', then the row vector $\mathbf{x}^{1}$' with size N, point of Λ', is obtained by $\mathbf{x}^{1}$' $= \mathbf{x}^1\mathbf{U} = \mathbf{b}^1\mathbf{B}^1$. So, the same transformation has to be performed on $\mathbf{y}^1$ before detection:

$$\mathbf{y}^{1\prime} = \mathbf{y}^1\mathbf{U} = \left(\mathbf{b}^1\mathbf{G}^1 + \mathbf{n}^{\mathrm{P}}\right)\mathbf{U} = \mathbf{b}^1\mathbf{B}^1 + \mathbf{n}^{1\prime} \tag{18}$$

We can easily show that $E[\mathbf{n}^{1\prime T}\mathbf{n}^{1\prime}] = N_0\ \mathbf{I}_N$. No further noise whitening is necessary. Elements of $\mathbf{b}^1$, obtained by sphere decoding of vector $\mathbf{y}^{1}$', are the non-fixed integer coordinates of the detected vector. Together with the fixed elements of $\mathbf{b}^0$, they form the detected vector $\hat{\mathbf{b}}$.

V. Simulation results

The sphere decoding has been tested in downlink on an indoor channel defined in [10], with a delay spread equal to 390 ns. All users have same power and transmit 16-QAM symbols on $L = 64$ sub-carriers over a 20 MHz bandwidth. With a HIPERLAN/2-like guard interval (25% of the OFDM symbol period), the obtained bit rate is 1 Mbit/s/user. Thus, by using a high spectral efficiency 16-QAM modulation, a total bit rate of 64 Mbits/s can be reached for full-load. The channel coefficients change for each transmitted symbol. We assume the power control being perfect *i.e.*, at each time *i*, the received symbol power is equal to the transmitted symbol power.

In Fig. 5, for half-load (32 users), the optimum performance of the sphere decoding is compared with the performance of three sub-optimum detection schemes: the single-user Minimum Mean Square Error Combining (MMSEC) [1], the multiuser Parallel Interference Cancellation (PIC) with 2 iterations including MMSEC and hard cancellation [2], and finally the multiuser Global Minimum Mean Square Error (GMMSE) [11] detector. Three detection schemes with sphere decoding are also tested. The Sphere Decoder A is the optimum detection scheme, whereas Sphere Decoders B and C include the projection described in section IV. Sphere decoding is performed in dimension $2K = 64$ for scheme B and in reduced dimension N for scheme C. For both schemes, we chose $\alpha_k = 0$ for $k = 1,\ldots,2K$. The Bit Error Rate (BER) averaged over all users is drawn versus the signal-to-noise ratio. The optimum multiuser performance is very close to the single-user one. The improvement with respect to multiuser GMMSE detection is 1.5 dB for a BER equal to 10^{-3}. The gap with MMSEC and PIC detection schemes is even higher. Both simplifications of the sphere decoding in versions B and C have no impact on performance even with the chosen α_k values, which maximise the number of projections.

Fig. 6 shows the signal-to-noise ratio (SNR) required to obtain an average BER equal to 10^{-3} versus the number of users. Sphere Decoder A is the optimum detector, whereas Sphere Decoder C includes projection on the lattice constellation and sphere decoding in the reduced dimension lattice ($\alpha_k = 0$ for $k = 1,\ldots,2K$). The gap in performance between optimum multiuser detection and the single user bound increases with the system load. This degradation equals 3 dB for 64 users (full-load), whereas the degradation of GMMSE equals 8.7 dB. The performance loss due to the restriction of sphere decoding to lattice Λ' is negligible with up to 56 users. Beyond this value, the performance degradation produced by the simplification sub-optimality grows sharply. However, the Sphere Decoder C performance still remains better than the GMMSE performance. GMMSE and Sphere Decoder C have same performance for full load, but intermediate performance results, closer to the optimum Sphere Decoder A, may be obtained for Sphere Decoder C by raising α_k values at the cost of a complexity increase.

Finally, simulations have shown that, with 48 users, the complexity of Sphere Decoder C was 14 times lower than the complexity of Sphere Decoder A at a SNR equal to 12 dB, for a negligible performance loss. It is worth pointing out that, with a full-loaded system employing 16-QAM modulation, an exhaustive ML search [2] would have required the computation of 2^{256} metrics to detect each vector $\mathbf{b}(i)$.

VI. Conclusions

We studied and simplified a low complexity ML multiuser detection for MC-CDMA systems based on their lattice representation. A lattice decoder is applied to optimally detect all users. Its complexity, independent of the modulation size and growing polynomially with the number of users, allows us to reach performance limits even for high loads and large modulations. The proposed simplifications through orthogonal projection of the received point further reduce the complexity for noisy received signals. No performance loss is observed for a 87% load, as compared to optimum detection. For higher loads, a performance degradation appears, which remains smaller than the degradation observed with the GMMSE detector.

References

[1] A. Chouly, A. Brajal, S. Jourdan: "Orthogonal multicarrier techniques applied to direct sequence spread spectrum CDMA systems," *GLOBECOM'93*, pp. 1723-1728, Nov. 1993.

[2] K. Fazel, L. Papke: "On the performance of convolutionally-coded CDMA/OFDM for mobile communication system," *PIMRC'93*, pp. 468-472, Sept. 1993.

[3] N. Yee, J.P. Linnartz, G. Fettweis: "Multicarrier CDMA in indoor wireless radio networks," *PIMRC'93*, pp. 109-113, Sept. 1993.

[4] S. Hara, R. Prasad: "Overview of multicarrier CDMA," *IEEE Comm. Mag.*, pp. 126-133, Dec. 1997.

[5] L. Brunel: "Optimum multiuser detection for MC-CDMA systems using sphere decoding," *PIMRC'01*, Oct. 2001.

[6] E. Viterbo, E. Biglieri: "A universal lattice decoder," *14ème Colloque GRETSI*, pp. 611-614, Sept. 1993.

[7] E. Viterbo, J. Boutros: "A universal lattice code decoder for fading channels," *IEEE Trans. on Inf. Theory*, vol. 45, pp. 1639-1642, July 1999.

[8] L. Brunel, J. Boutros: "Euclidean space lattice decoding for joint detection in CDMA systems," *ITW'99*, p. 129, June 1999.

[9] J.H. Conway, N.J. Sloane: *Sphere packings, lattices and groups*, 1998, Springer-Verlag, New York.

[10] J. Medbo: "Channel Models for HIPERLAN/2 in Different Indoor Scenarios," *ETSI BRAN doc. 3ERI085b*, Mar. 1998.

[11] J.F. Hélard, J.Y. Baudais, J. Citerne: "Linear MMSE detection technique for MC-CDMA," *Electron. Letters*, pp. 665-666, Mar. 2000.

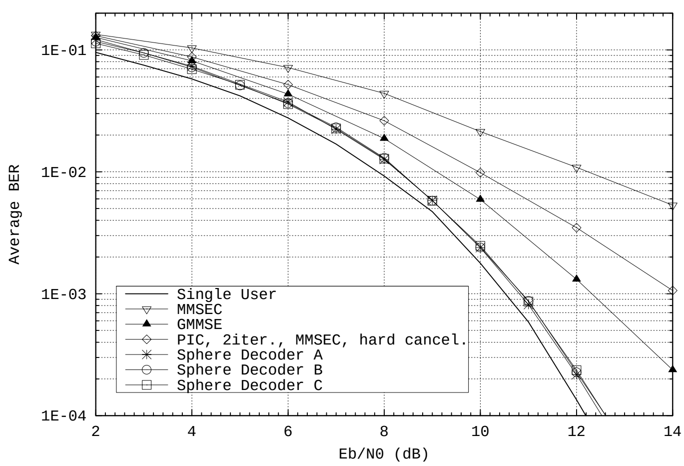

Fig. 5: Comparison of various detection techniques, 32 users.

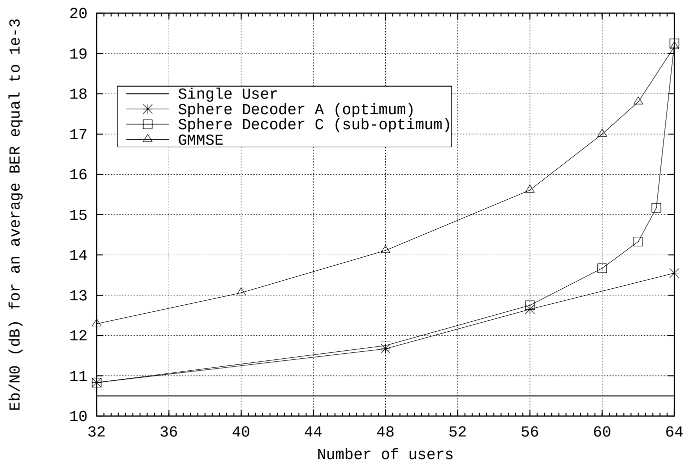

Fig. 6: Comparison of various detection techniques, BER=10^{-3}.