LINEAR PRECODING UNDER ITERATIVE PROCESSING FOR MULTIPLE ANTENNA CHANNELS

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ABSTRACT

We propose a universal linear precoder for bit-interleaved coded QAM modulations transmitted over frequency non-selective multiple antenna channels. The precoder is based on cyclotomic multi-dimensional rotations and does not require the knowledge of the channel matrix at the transmitter side. The precoding performance analysis is made via the genie method by assuming a receiver with iterative APP detection. We also describe a new singleton bound on the diversity order that takes into account the time-spreading factor of the precoder. This new bound is indispensable to select the precoder size yielding the maximum diversity order while minimizing the detector's complexity.

1. INTRODUCTION

It is proven that using multiple antennas in both transmission and reception is an excellent solution to enhance the diversity order and the information rate of digital communication systems [1][2]. When fading is white and frequency non-selective, the receive diversity brought by the number of receive antennas is always collected without difficulty. The multiple transmit antennas yield the double advantage of increasing the data rate and augmenting the available transmit diversity. A different source of diversity is given by the number of different channel states observed during one codeword time length. Multi-dimensional rotations [3] and space-time block codes[4] have been designed to produce transmit diversity by spreading symbols in space and time before transmission. We present a modification of the cyclotomic rotations that insures a pre-determined diversity at the output of the detector at the last iteration of an iterative joint detection and decoding. This diversity factor is dependent of the channel parameters and the linear spreading factor. When using a bit-interleaved coded modulation (BICM [5]) over a block-fading channel, the diversity collected by the error-correcting code is limited by the singleton bound [6] of the BICM. We propose a modified singleton bound that takes into account the linear precoding applied at the transmitter side, and we adapt the linear space-time spreading factor to boost the singleton bound up to the diversity order given by the outage capacity. The diversity exploitation complexity is then dispatched between the detector and the decoder of the BICM receiver.

Section 2 presents the system model and parameters, then, Section 3 makes the presentation of linear precoders [7] available for any channel configuration and achieving a well pre-designed diversity. Section 4 is devoted to the modified singleton bound that includes the influence of linear precoding. The minimal spreading factor that exhibits full diversity is computed. Finally, computer simulation results are shown to illustrate the theoretical results.

2. SYSTEM MODEL AND PARAMETERS

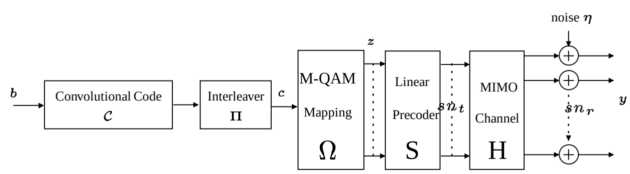

Fig. 1. Bit-interleaved coded multiple antenna transmitter.

A BICM model is presented on Fig. 1: A rate R_c binary error-correcting code $\mathcal{C}$ converts the information word b into a codeword c. The coded bits are then interleaved by a semi-deterministic interleaver Π presented in [7], mapped into 2^m-QAM symbols, and then linearly precoded before transmission on the MIMO channel. We consider a frequency non-selective channel with n_t transmit antennas, n_r receive antennas, perfect channel state information and coherent detection at the receiver side. The coefficients of the $n_t \times n_r$ channel matrix H_ℓ at time index ℓ are independent complex Gaussian distributed random variables with unit variance. The channel is block-fading, i.e., there are n_c different channel realizations($\ell = 1 \ldots n_c$). Each channel state is constant over L/n_c symbol time periods, where L is the codeword time length. The $sn_t \times sn_t$ precoding matrix S spreads sn_t QAM symbols over s time periods. The $sn_t \times sn_r$ extended channel matrix used L/s times is

$$H = diag\{ \underbrace{H_1, \ldots, H_1}_{s/n_c}, \ldots, H_{n_c}, \ldots, H_{n_c}\} \tag{1}$$

Now, we can write the channel input-output relation:

$$y = x + \eta = zSH + \eta \qquad (2)$$

where $z \in \Omega$, $\Omega = (2^m - \text{QAM})^{sn_t}$, $y \in \mathbb{C}^{sn_r}$ and each receive antenna is perturbed by an additive zero-mean white complex Gaussian noise η. The spectral efficiency of the transmission system is $R_c m n_t$ bits per channel use. At the receiver side, a classical iterative joint detection and decoding is made, where a soft-input soft-output (SISO) detector converts the received observation y into a posteriori probabilities. A priori probabilities (extrinsics) are exchanged between the SISO detector and the SISO decoder of $\mathcal{C}$.

3. LINEAR PRECODING FOR MIMO CHANNELS UNDER ITERATIVE DETECTION

As mentioned above, the receive diversity n_r is naturally collected by the detector. We assume a quasi perfect a priori feedback from the decoder (genie condition, see [7]). The genie situation occurs in an ideal system after an infinite number of detection/decoding iterations under some conditions on the code structure and the signal-to-noise ratio value. In this case, only two points of the multi-dimensional constellation Ω are selected to compute the extrinsic probability of a bit. The Euclidean distance between these two points is Nakagami n_r-distributed and thus the diversity order carried by a priori probabilities fed to the SISO decoder is n_r. The system has an intrinsic transmit diversity equal to $n_t n_c$. The maximum diversity that can be recovered by well-designed coding $\mathcal{C}$ and precoding S is $n_t n_c n_r$, which is equal to the number of distinct coefficients h_{ij} observed by a codeword, $H = [h_{ij}]$. The precoder role is to enhance the diversity embedded in the extrinsic probabilities at the detector output before decoding.
Linear precoders have been proposed in [7] for any n_t, n_c and $s \leq n_c n_t$ leading to a diversity sn_r at the detector output. It was shown that the linear precoder should satisfy the following properties under iterative APP decoding of the space-time BICM: 1- The $1/n_c$-th parts of the rows of a $sn_t \times sn_t$ spreading matrix have the same Euclidean norm, 2- In each $1/n_c$-th part, the n_c/s-th parts are orthogonal and have the same Euclidean norm. Any matrix that satisfies the above properties leads to a diversity order equal to sn_r ($s \leq n_t n_c$)at the output of the detector at the last iteration of a converging iterative joint detection and decoding. However, all matrices verifying such conditions do not lead to the same convergence behaviour. It is well known that a first iteration with good performance, close to the maximum likelihood (ML) one, leads to fast convergence. We need linear precoding matrices having good ML performance and existing for any value of sn_t. Cyclotomic rotations satisfy both conditions [3] and have unity norm coefficients, which allows to satisfy the norm condition presented above. Let S be a $sn_t \times sn_t$ cyclotomic rotation, the coefficients of which are

$$S_{i,\ell} = \exp\left(2j\pi i \left[\frac{1}{\Phi^{-1}(2sn_t)} + \frac{\ell}{sn_t}\right]\right) \qquad (3)$$

Here, ϕ is the Euler totient function, $j = \sqrt{-1}$,$i = 0 \ldots sn_t - 1$, $\ell = 0 \ldots sn_t - 1$. As this cyclotomic rotation matrix does not satisfy the properties leading to maximum diversity with spreading factor s, we apply multiplicative correction terms $e^{2j\pi\alpha_{i,w,t,v}}$ to S coefficients, $i = 0 \ldots sn_t - 1$, $w = 0 \ldots n_c - 1$, $t = 0 \ldots s/n_c - 1$, $v = 0 \ldots n_t - 1$ and $v + (ws/n_c + t)n_t = \ell$.

$$\begin{aligned} S_{i,w,t,v} = &\tfrac{1}{\sqrt{sn_t}} \exp\Big(2j\pi \Big[i\Big(\tfrac{1}{\Phi^{-1}(2sn_t)} + \\ &\tfrac{v}{sn_t} + \tfrac{t}{s} + \tfrac{w}{c}\Big) + \alpha_{i,w,t,v}\Big]\Big) \end{aligned} \qquad (4)$$

The first condition on $\alpha_{u,w,t,v}$ is that S should stay a rotation for convergence considerations, i.e., $\forall i, i'$

$$\begin{aligned} &\forall i, i', \textstyle\sum_{w,v,t} S_{i,w,t,v} S^*_{i',w,t,v} = \delta(i,i') \\ &\Leftrightarrow \textstyle\sum_{w,v,t} \tfrac{1}{\sqrt{sn_t}} \exp\Big(2j\pi\Big[(i-i')\Big(\tfrac{1}{\Phi^{-1}(2sn_t)} \\ &\quad + \tfrac{v}{sn_t} + \tfrac{t}{s} + \tfrac{w}{c}\Big) + \alpha_{i,w,t,v} - \alpha_{i',w,t,v}\Big]\Big) = \delta(i,i') \end{aligned} \qquad (5)$$

where $\delta(i,i') = 1$ if $i' = i$, else 0. The rotation condition is satisfied if $\forall (i,i')$, $\alpha_{i,w,t,v} = \alpha_{i',w,t,v}$. Since each coefficient has unity norm, the norm properties are satisfied. The orthogonality between sub-parts of the rows leads to

$$\frac{\exp\left(2j\pi i \frac{t-t'}{s}\right)}{\sqrt{n_t}} \sum_{v=0}^{n_t-1} \exp\left(2j\pi(\alpha_{i,w,t,v} - \alpha_{i,w,t',v})\right) = \delta(t,t') \qquad (6)$$

The equality is satisfied if $\exp(2j\pi\alpha_{i,w,t,v})$ is chosen equal to the (t,v) coefficient of a $n_t \times n_t$ cyclotomic rotation, i.e.,

$$\alpha_{i,w,t,v} = t\left(\frac{1}{\Phi^{-1}(2n_t)} + \frac{v}{n_t}\right) \qquad (7)$$

Finally, matrix S is a modified cyclotomic rotation and its entries are

$$\begin{aligned} S_{i,v+(ws/n_c+t)n_t} = &\tfrac{1}{\sqrt{sn_t}} \exp\Big(j2\pi\Big[i\Big(\tfrac{1}{\phi^{-1}(2sn_t)} + \\ &\tfrac{v}{sn_t} + \tfrac{t}{s} + \tfrac{w}{n_c}\Big) + t\Big(\tfrac{1}{\phi^{-1}(2n_t)} + \tfrac{v}{n_t}\Big)\Big]\Big) \end{aligned} \qquad (8)$$

where $i = 0 \ldots sn_t - 1$, $w = 0 \ldots n_c - 1$, $t = 0 \ldots s/n_c - 1$, and $v = 0 \ldots n_t - 1$.

4. MODIFIED SINGLETON BOUND

The full diversity $n_t n_c n_r$ is collected by the detector when $s = n_t n_c$, but unfortunately, the APP signal detection has an exponential complexity in s. On the other hand, the BICM channel decoder is also capable of collecting a large amount of diversity, but the latter is still limited by the singleton bound [6]. Hence, the lowest complexity solution that reaches full diversity is to draw advantage of the whole channel code diversity and recover the remaining diversity by linear precoding. The best way to choose the spreading factor s of a cyclotomic rotation is given by the modified singleton bound described hereafter.

Now, let us examine the Nakagami distributions at the decoder input.The $\mathcal{C}$ decoder recombines via an APP decoding algorithm the extrinsic probabilities produced by the MIMO detector, or equivalently, their Nakagami distributed Euclidean distances when the genie is activated. The Nakagami distribution has order sn_r. Recall that $s = 1$ when symbols are not precoded. In the latter situation, it is trivial to show that the number of independent Nakagami laws at the decoder input is $n_c n_t$. In the most interesting situation, i.e., s-spread linear precoding with $s > 1$, it is easy to show that the number of independent Nakagami laws given to the decoder is $N = \lfloor \frac{n_c n_t}{s} \rfloor$. This integer N is the best diversity multiplication factor to be collected by $\mathcal{C}$.

The length of a $\mathcal{C}$ codeword is Lmn_t binary elements. Let us group Lmn_t/N bits into one non-binary symbol. Now, $\mathcal{C}$ is a length N code built on an alphabet of size $2^{Lmn_t/N}$. The singleton bound on the minimum Hamming distance of the non-binary $\mathcal{C}$ becomes $D_H \leq N - \lceil NR_c \rceil + 1$. Multiplying the previous inequality with the Nakagami law order sn_r yields the maximum achievable diversity order d_{max} after decoding,

$$d_{max} \leq sn_r \left\lfloor \left\lfloor \frac{n_c n_t}{s} \right\rfloor (1 - R_c) + 1 \right\rfloor \tag{9}$$

where d_{max} is an integer. Finally, since d_{max} is upper-bounded by the channel intrinsic diversity and the minimum Hamming distance d_H of the binary code, we can write

$$d_{max} \leq \min \left(sn_r \left\lfloor \left\lfloor \frac{n_c n_t}{s} \right\rfloor (1 - R_c) + 1 \right\rfloor ; n_t n_c n_r ; sn_r d_H \right) \tag{10}$$

If d_H is not a limiting factor (choose$\mathcal{C}$ accordingly),we can select the value of s that leads to a modified singleton bound greater than or equal to $n_t n_c n_r$. To do so, two necessary conditions must be satisfied: $n_c n_t$ is a multiple of s and $s \geq R_c n_c n_t$.

Proposition: Considering a BICM with a rate R_c binary error-correcting code on a $n_t \times n_r$ MIMO channel with n_c distinct channel states per codeword, the spreading factor s of a linear precoder must divide $n_t n_c$ and must satisfy $s \geq R_c n_c n_t$ in order to achieve the full diversity $n_t n_c n_r$.

The smallest integer s_{opt} satisfying the above proposition minimizes the detector's complexity. If $R_c > 1/2$, then $s_{opt} = n_c n_t$ which involves the highest complexity. If $R_c \leq 1/(n_c n_t)$, no linear precoder is necessary.

Table 1 and 2 show the diversity order derived from the singleton bound versus s and n_t,for $n_c = 1$ and $n_c = 2$, respectively. The values in bold indicate full diversity configurations. For example, in Table 1, $n_c = 1$, for $n_t = 4$,$s = 2$ is a better choice than $s = 4$ since it leads to an identical diversity order with a lower complexity.

$n_t \diagdown s$	1	2	3	4	5	6	7	8
1	**1**							
2	**2**	**2**						
3	2	2	**3**					
4	3	**4**	3	**4**				
5	3	4	3	4	**5**			
6	4	4	**6**	4	5	**6**		
7	4	4	6	4	5	6	**7**	
8	5	6	6	**8**	5	6	7	**8**

Table 1. $R_c = 1/2, n_r = 1$ and $n_c = 1$.

$n_t \diagdown s$	1	2	3	4	5	6	7	8
1	**2**	**2**						
2	3	**4**	3	**4**				
3	4	4	**6**	4	5	**6**		
4	5	6	6	**8**	5	6	7	**8**
5	6	6	6	8	**10**	6	7	8
6	7	8	9	8	10	**12**	7	8
7	8	8	9	8	10	12	**14**	8
8	9	10	9	12	10	12	14	**16**

Table 2. $R_c = 1/2, n_r = 1, n_c = 2$.

5. SIMULATION RESULTS

Computer simulations have been made to illustrate the bounds on the achievable diversity. The space-time optimized interleaver presented in [7] has been used in Monte Carlo simulations of both figures. The binary code is the terminated classical non-recursive non-systematic (7,5) convolutional code. The codeword length is 256 bits. Simple BPSK is transmitted on the MIMO channel, $m = 1$. Fig. 2 illustrates the frame-error rate performance on a 2×1 MIMO channel with 2 distinct states per codeword. The full diversity order is 4. The singleton bound order without linear precoding is 3 and the achieved one is 2. The latter is raised up to 4 with $s = 2$ linear precoding. Fig. 3 shows a similar scenario with $n_t = 4$ and $n_c = 1$ on a 4×1 MIMO channel. Again, as predicted by the singleton bound, the full diversity order 4 is achieved via $s = 2$ linear precoding. The simulations have been conducted with $n_r = 1$ in order to magnify the effect of diversity increase.

6. CONCLUSIONS

We constructed a linear precoder ensuring a pre-determined diversity at the output of the detector at the last iteration of

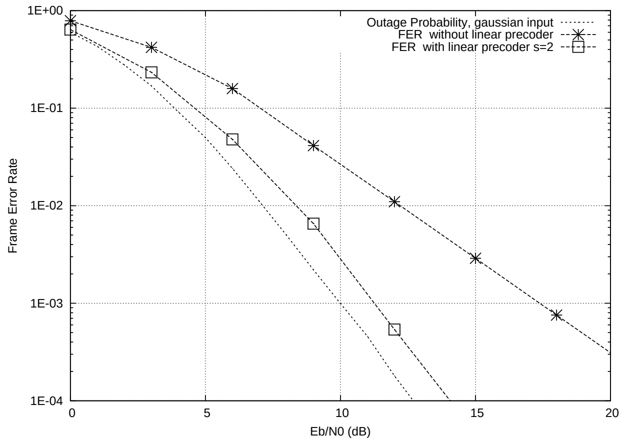

Fig. 2. Performance of a 4-state rate 1/2 convolutional code, 2×1 MIMO block-fading channel, $n_c = 2$ states, 3 detection/decoding iterations.

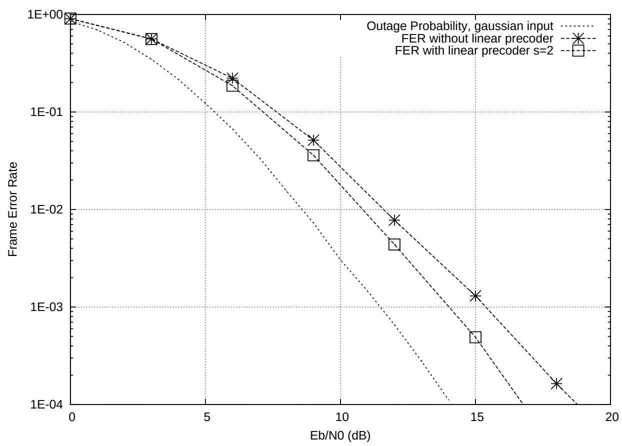

Fig. 3. Performance of a 4-state rate 1/2 convolutional, 4×1 MIMO quasi-static channel, $n_c = 1$ state, 3 detection/decoding iterations.

an iterative MIMO BICM receiver. We described a modified singleton bound on the diversity order that takes into account the linear precoding in a space-time transmitter. The diversity order of these bit-interleaved coded modulations is guaranteed by a judicious choice of both interleaving and precoding matrix. It is shown how to choose the time-spreading factor of the linear precoder in order to attain the maximum diversity order while minimizing the detector's complexity.

7. REFERENCES

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