Optimal linear precoding for BICM over MIMO channels

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Abstract — **We present a linear precoding solution to achieve full diversity with iteratively decoded bit-interleaved coded modulation on multiple antenna channels while minimizing the detection complexity.**

I. Introduction

Bit-interleaved coded modulations (BICM [1]) with iterative joint detection and decoding on block fading multiple antenna channels do not always exhibit full diversity. The achieved diversity is limited by the singleton bound [2]. We use a modification of the cyclotomic rotations for linear precoding that insures a pre-determined diversity at the output of the detector, at the final iteration of an iterative joint detection and decoding procedure [3]. We also derive the singleton bound that takes into account the space-time precoding. The precoder size is adapted to boost the singleton bound up to the diversity order given by the outage capacity. The complexity is minimized by distributing the diversity exploitation between the detector and the decoder in the BICM receiver.

II. System Model

We consider a frequency non-selective channel with n_t transmit antennas, n_r receive antennas, perfect channel state information and coherent detection at the receiver side. The channel is block-fading, i.e., there are n_c different channel states. The coefficients of the $n_t \times n_r$ channel matrix H_ℓ, $\ell = 1 \ldots n_c$, are i.i.d complex gaussians $\mathcal{N}_{\mathbb{C}}(0,1)$. The BICM transmitter includes a rate R_c binary error-correcting code $\mathcal{C}$, a binary optimized interleaver [3], and a QAM mapper. The latter is followed by a $sn_t \times sn_t$ precoding matrix S that spreads sn_t QAM symbols over s time periods before transmission on the multiple antenna (MIMO) channel.

Each channel state is constant over L/n_c symbol time periods, where L is the codeword time length. We can write the channel input-output relation as $y = x + \eta = zSH + \eta$, where H is a block diagonal matrix with s blocks H_ℓ, $z \in \Omega$, $\Omega = (2^m - \text{QAM})^{sn_t}$, $y \in \mathbb{C}^{sn_r}$ and each receive antenna is perturbed by an additive zero-mean white complex Gaussian noise η. The spectral efficiency is $R_c m n_t$ bits per channel use. At the receiver side, a classical iterative joint detection and decoding is performed [3].

III. Linear precoding for MIMO channels under iterative detection

The receive diversity n_r is naturally collected by the detector. We assume a quasi perfect a priori information fedback from the decoder to the detector (the genie method). It was shown in [3] that a linear precoder achieving a diversity order sn_r with maximum coding gain must satisfy the following conditions under iterative APP decoding of the space-time BICM: 1- The $1/n_c$-th parts of the rows in the $sn_t \times sn_t$ precoding matrix have the same Euclidean norm, 2- In each $1/n_c$-th part, the n_c/s-th parts are orthogonal and have the same Euclidean norm. We need linear precoding matrices exhibiting good performance at the 1rst iteration and available for any value of sn_t. Cyclotomic rotations are good candidates, they satisfy condition 1 and we modify them to obtain the following coefficients satisfying condition 2.

$$S_{i,v+(ws/n_c+t)n_t} =$$

$$\frac{1}{\sqrt{sn_t}} \exp\left(j2\pi\left[i\left(\frac{1}{\psi(2sn_t)} + \frac{v}{sn_t} + \frac{t}{s} + \frac{w}{n_c}\right) + t\left(\frac{1}{\psi(2n_t)} + \frac{v}{n_t}\right)\right]\right)$$

where ψ is the reciprocal of the Euler's ϕ totient function, and $0 \le i < sn_t, 0 \le w < n_c, 0 \le t < s/n_c, 0 \le v < n_t$.

IV. Singleton bound with precoding

The full diversity $n_t n_c n_r$ is collected by the detector when $s = n_t n_c$. Unfortunately, soft-output detection has a complexity increasing with s (at most exponentially). On the other hand, the channel decoder is also capable of collecting a large amount of diversity, but the latter is still limited by the singleton bound. The channel decoder combines sn_r-order Nakagami distributed Euclidean distances when the genie is activated. The number of independent Nakagami laws given to the decoder is $N = \lfloor \frac{n_c n_t}{s} \rfloor$. The code $\mathcal{C}$ can be seen as a length N non-binary code with alphabet size $2^{Lmn_t/N}$. The singleton bound on the minimum Hamming distance of the non-binary $\mathcal{C}$ becomes $D_H \le N - \lceil NR_c \rceil + 1$. Finally, the maximum achievable diversity order d_{max} is upper-bounded by the modified singleton bound, the channel intrinsic diversity and the minimum distance d_H of the binary code :

$$d_{max} \le \min\left(sn_r \left\lfloor \left\lfloor \frac{n_c n_t}{s} \right\rfloor (1 - R_c) + 1 \right\rfloor ; n_t n_c n_r ; sn_r d_H\right)$$

If d_H is not a limiting factor (choose $\mathcal{C}$ accordingly), we can select the value of s that leads to a modified singleton bound greater than or equal to $n_t n_c n_r$.

Proposition : Considering a BICM with a rate R_c binary error-correcting code on a $n_t \times n_r$ MIMO channel with n_c distinct channel states per codeword, the spreading factor s of a linear precoder must divide $n_t n_c$ and must satisfy $s \ge R_c n_c n_t$ in order to achieve the full diversity $n_t n_c n_r$. For example, $n_c = 1$ and $n_t = 4$, $s = 2$ is a better choice than $s = 4$ since it leads to full diversity order with a lower detection complexity.

References

[1] G. Caire, G. Taricco, E. Biglieri, "Bit-interleaved coded modulation," *IEEE Trans. on Inf. Theory*, vol. 44, no. 3, pp. 927-946, May 1998.

[2] R. Knopp, P. Humblet, "On coding for block fading channels," *IEEE Trans. on Inf. Theory*, vol. 46, no. 1, pp. 189-205, Jan. 2000.

[3] J. Boutros, N. Gresset, L. Brunel, "Turbo coding and decoding for multiple antenna channels," *Int. Symp. on Turbo Codes*, Brest, Sept. 2003. Available at *http://www.enst.fr/∼gresset*.