A Pre-FFT OFDM Adaptive Antenna Array with Eigenvector Combining

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Abstract— **This paper proposes a novel pre-FFT type OFDM adaptive array antenna called "Eigenvector Combining." The eigenvector combining is a realization of a post-FFT type OFDM adaptive array antenna through a pre-FFT signal processing, so it can achieve excellent performance with less computational complexity and shorter training symbols. Numerical results demonstrate that the proposed eigenvector combining shows excellent bit error rate performance close to the lower bound just with two training symbols.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM), which is a pre-distortion or an equalization technique at transmit side in a sense, is an efficient technique for high-speed digital transmission over severe multipath fading channels[1], and recently has been considered to be a promising technique for next generation mobile communications systems[2]. OFDM scheme inserts a guard interval in every OFDM symbol so as to be robust to delayed signals within the guard interval, however, once delayed signals beyond the guard interval (namely, co-channel interfering signals) are introduced in a channel with larger delay spread, inter-symbol (co-channel) interference causes a severe degradation in the transmission performance.

To maintain high-speed reliable wireless communications systems, the use of multiple antennas at receive side has been considered as an effective tool not only for gain enhancement, increased spectral efficiency[3] but also for interference suppression[4]. Although a *post*-fast Fourier transform (FFT) subcarrier-by-subcarrier combining OFDM adaptive array antenna is optimum in terms of maximizing signal-to-interference-and-noise power ratio (*SINR*), it requires the increased number of FFT processors and heavy computations, which increase with the number of antennas and subcarriers, and a quite long training signal[5]. On the other hand, a *pre*-FFT type OFDM adaptive array antenna, which requires only one FFT processor, can drastically reduce the computational complexity by tolerating some performance degradation[6], [7].

This paper proposes a novel pre-FFT type OFDM adaptive array antenna called "Eigenvector Combining[8]." The eigenvector combining is a realization of a post-FFT type OFDM adaptive array antenna through a pre-FFT signal processing[9], so it has the characteristics of the two types of the array antennas, namely, excellent performance inherent in the post-FFT type and less computational complexity and shorter training symbols proper to the pre-FFT type.

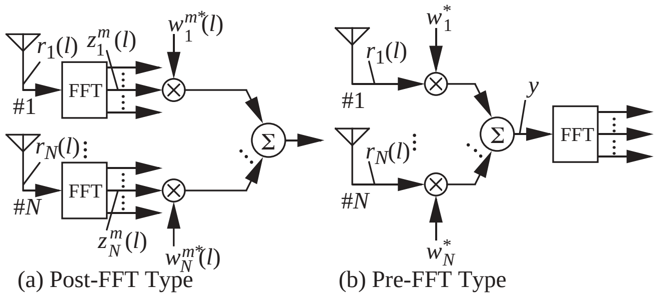


Fig. 1. OFDM adaptive array antennas.

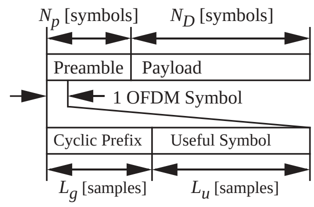


Fig. 2. Signal burst format.

II. System Model

Figures 1 (a) and (b) shows the block diagrams of a post-FFT type OFDM adaptive array antenna and a pre-FFT type OFDM adaptive array antenna, respectively. Each antenna is equipped with N antenna elements.

Figure 2 shows a signal burst format. The signal burst is composed of preamble and payload. The preamble is composed of N_p training OFDM symbols and N_D data OFDM symbols, and one OFDM symbol is composed of L_g sample-long cyclic prefix and L_u sample-long useful symbol.

III. Pros and Cons of Pre- and Post-FFT Type Array Antennas

First, let us pay attention to the post-FFT array antenna as shown in Fig.1 (a). The received signal vector $\mathbf{r}$ $(N \times 1)$ is composed of the desired signal vector $\mathbf{x}$ $(N \times 1)$, the interfering signal vector $\mathbf{i}$ $(N \times 1)$ and the noise vector $\mathbf{n}$ $(N \times 1)$:

$$\mathbf{r} = \mathbf{x} + \mathbf{i} + \mathbf{n}. \quad (1)$$

$\mathbf{x}$, $\mathbf{i}$ and $\mathbf{n}$ are mutually uncorrelated:

$$\mathbf{r} = [r_1(l), \cdots, r_N(l)]^T, \quad (2)$$

$$\mathbf{x} = [x_1(l), \cdots, x_N(l)]^T, \quad (3)$$

$$\mathbf{i} = [i_1(l), \cdots, i_N(l)]^T, \quad (4)$$

$$\mathbf{n} = [n_1(l), \cdots, n_N(l)]^T, \quad (5)$$

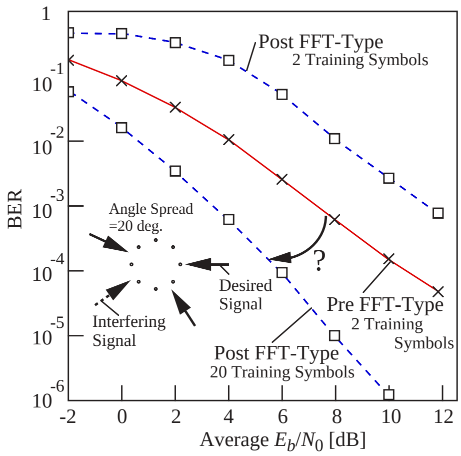


Fig. 3. BER of post- and pre-FFT type array antennas.

where T and l denote transpose and sampling index, respectively.

Define $\mathbf{z}^m$ as the FFT output vector ($N \times 1$) for the m-th subcarrier:

$$\mathbf{z}^m = [z_1^m(l), \cdots, z_N^m(l)]^T. \tag{6}$$

The weight control criterion for the post-FFT array antenna is given by

$$\text{minimize} \;\; E\left[|p^m(l) - \mathbf{w}^{mH}\mathbf{z}^m|^2\right] \quad (m = 1, \cdots, M), \tag{7}$$

where $E[\cdot]$ denotes expectation, H denotes Hermitian transpose, M is the number of subcarriers, $p^m(l)$ is the training symbol at the m-th subcarrier and $\mathbf{w}^m$ is the array weight vector ($N \times 1$):

$$\mathbf{w}^m = [w_1^m(l), \cdots, w_N^m(l)]^T. \tag{8}$$

Equation (7) can be solved by well known adaptive algorithms. However, the conventional algorithms require at least several tens of symbols for good convergence, in other words, *they require several tens of training OFDM symbols in the preamble* (note that one training OFDM symbol gives only one reference point at each subcarrier).

Figure 3 shows the BER (Bit Error Rate) of a post-FFT type array antenna (with $M = 52$, $N_D = 10$, $L_g = 16$ and $L_u = 64$) for the case of three desired signals and one interfering signal (see the spatio-temporal model in the same figure, or see Section VII). Here, the Sample Matrix Inversion (SMI) algorithm is used for array weight calculation[4], so the computational complexity is $O(N_p^3 \times (L_g + L_u)^3 \times M)$. When setting $N_p = 2$, the BER is poor, on the other hand, when setting $N_p = 20$, the BER is excellent, but the burst efficiency is 0.33 (=10/(20+10)).

Next, let us pay attention to the pre-FFT array antenna as shown in Fig.1 (b). The pre-FFT type weight control is based on *not OFDM symbol but sample*-driven algorithm, so two OFDM symbol-long preamble ($N_p = 2$, $N_p \times (L_p + L_u) =$ 160 samples) is long enough to get good convergence. Figure 3 also shows the BER of a pre-FFT array antenna (see Section IV for the detail algorithm). Here, the SMI algorithm is used for array weight calculation, so the computational complexity is $O(N_p^3 \times (L_g + L_u)^3)$. As compared with the post-FFT array antenna with the same preamble length, the pre-FFT type can achieve a much better BER performance, whereas its performance is still inferior to that of the post-FFT type with enough preamble length. Therefore, a key issue in the paper is *how to make the performance of a pre-FFT array antenna be close to that of the post-FFT array antenna, keeping the shorter preamble length and the less computational complexity.*

IV. Principle of Pre-FFT Type Array Antenna

Define the OFDM signal vector ($L_g \times 1$) and the channel impulse response vector ($L_g \times 1$) at the n-th antenna element as

$$\mathbf{s} = [s(1), \cdots, s(-L_g)]^T, \tag{9}$$
$$\mathbf{h}_n = [h_{n1}, \cdots, h_{nL_g}]^T, \tag{10}$$

where $s(l)$ is the transmitted OFDM signal sampled at the l-th sampling instant. In addition, define the channel impulse response matrix ($N \times L_g$) as

$$\mathbf{H} = [\mathbf{h}_1, \cdots, \mathbf{h}_N]^T. \tag{11}$$

With Eqs.(9) and (11), $\mathbf{x}$ is written as

$$\mathbf{r} = \mathbf{Hs}. \tag{12}$$

The FFT input is written as

$$y = \mathbf{w}^H\mathbf{r}, \tag{13}$$

where $\mathbf{w}$ is the array weight vector ($N \times 1$):

$$\mathbf{w} = [w_1, \cdots, w_N]^T. \tag{14}$$

The *SINR* of the FFT input can be calculated as[7]

$$SINR = \frac{\mathbf{w}^H\mathbf{C}_x\mathbf{w}}{\mathbf{w}^H\mathbf{C}_{rec}\mathbf{w}}. \tag{15}$$

$\mathbf{C}_x$ and $\mathbf{C}_{rec}$ are the correlation matrices ($N \times N$) of the desired signal and the received signal:

$$\mathbf{C}_x = E[\mathbf{xx}^H] = E[\mathbf{Hss}^H\mathbf{H}^H] = \sigma_s^2\mathbf{HH}^H, \tag{16}$$
$$\mathbf{C}_{rec} = E[\mathbf{rr}^H], \tag{17}$$

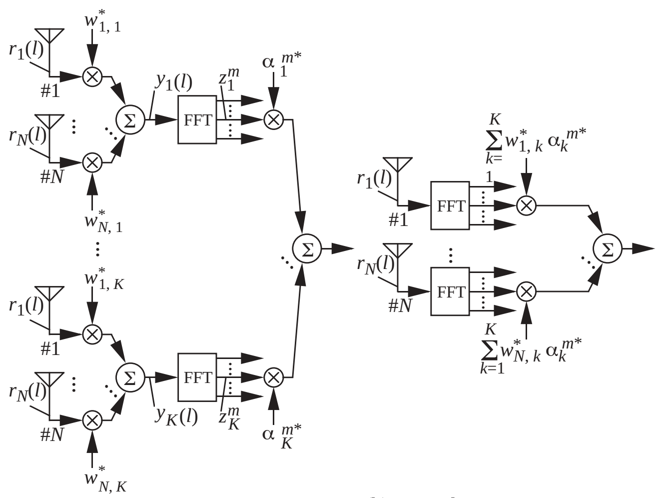
where σ_s^2 is the average power of the OFDM signal. For derivation of Eq.(16), the following important property of OFDM signal is used:

$$E[\mathbf{ss}^H] = \sigma_s^2\mathbf{I}_{L_g}, \tag{18}$$

where $\mathbf{I}_{L_g}$ is the identity matrix ($L_g \times L_g$).

The weight control criterion for the pre-FFT array antenna:

$$\text{maximize} \;\; SINR = \frac{\mathbf{w}^H\mathbf{C}_x\mathbf{w}}{\mathbf{w}^H\mathbf{C}_{rec}\mathbf{w}} \tag{19}$$



(a) Proposed Eigenvector Combining
(b) Equivalent Post-FFT Type Representation

Fig. 4. Eigenvector combining.

leads to the solution of the following generalized eigenvalue problem:

$$\begin{array}{ccc} \text{find} & \mathbf{w} & \text{which satisfies} \\ \mathbf{C}_x\mathbf{w} & = & \lambda_{max}\mathbf{C}_{rec}\mathbf{w}, \end{array} \tag{20}$$

where λ_{max} is the first largest generalized eigenvalue. When setting the array weights so as to satisfy Eq.(20), the largest *SINR* given by λ_{max} is realizable at the FFT input.

V. Eigenvector Combining

When the angle spread of received signals is small enough, the first largest generalized eigenvalue is dominant, so the pre-FFT array antenna can utilize almost all part of the received signal power. In this case, there is no problem with the pre-FFT array antenna. However, when the angle spread becomes large, the other eigenvalues such as the second and third largest ones become larger (namely, their values become comparable with the value of the first largest one), so the pre-FFT array antenna shows *a power utilization inefficiency* in the transmission performance. *Utilization of the array weights associated with the other larger eigenvalues can improve the transmission performance.* This is the principle of the eigenvector combining.

Equation (20) can be rewritten as

$$\mathbf{C}_x\mathbf{w}_k = \lambda_k\mathbf{C}_{rec}\mathbf{w}_k \ \ (k = 1, \cdots, K), \tag{21}$$

where there are assumed to be K non-zero generalized eigenvalues in the decreasing order, namely, $\lambda_1 = \lambda_{max} \geq \lambda_2 \geq \cdots \lambda_K$.

Figure 4 (a) shows the structure of the proposed pre-FFT array antenna with eigenvector combining, which virtually has K sets of pre-FFT array antennas. The *SINR* of the k-th pre-FFT array antenna is given by λ_k, where the power of interfering signals can be well suppressed. Therefore, the K signals after being combined with a set of $\mathbf{w}_k \ \ (k = 1, \cdots, K)$ are coherently combined on *subcarrier-by-subcarrier* basis (also after FFT operation) in a maximum ratio combining (MRC) manner. Note that to perform the MRC properly, a linear constraint needs to be imposed to Eq.(21):

$$\text{subject to } \ |\mathbf{w}_k| = 1. \tag{22}$$

The signal combined by $\mathbf{w}_k$ at the k-th virtual array branch is written as

$$y_k(l) = \mathbf{w}_k^H\mathbf{r}(l) \ \ (l = L_g, \cdots, L_g + L_u) \tag{23}$$

and the FFT input vector ($L_u \times 1$) at the k-th virtual branch is written as

$$\begin{aligned} \mathbf{y}_k &= [y_k(L_g), \cdots, y_k(L_g + L_u)]^T \\ &= \left(\mathbf{w}_k^H[\mathbf{r}(L_g), \cdots, \mathbf{r}(L_g + L_u)]\right)^T \\ &= \left(\mathbf{w}_k^H\mathbf{R}\right)^T, \end{aligned} \tag{24}$$

where $\mathbf{R}$ is the received signal matrix ($N \times L_u$).

Defining $\mathbf{d}^m = [1, e^{-j2\pi m/L_u}, \cdots, e^{-j2\pi(L_u-1)m/L_u}]^T$ as the L_u-point FFT vector ($L_u \times 1$) for the m-th frequency (subcarrier) component, the m-th subcarrier output at the k-th virtual array branch can be written as

$$z_k^m = \mathbf{y}_k^T\mathbf{d}^m. \tag{25}$$

On the other hand, the complex envelope for the m-th subcarrier component at the k-th virtual array branch is given by

$$\alpha_k^m = \mathbf{w}_k^H\mathbf{H}'\mathbf{d}^m, \tag{26}$$

where $\mathbf{H}'$ is the impulse response matrix ($N \times L_u$) obtained by $(L_u - L_g)$-zero padding to the row vector in $\mathbf{H}$. Therefore, the MRC-combined m-th final subcarrier output is written as

$$\begin{aligned} v^m &= \sum_{k=1}^{K}\alpha_k^{m*}z_k^m \\ &= \mathbf{d}^{mH}\mathbf{H}'^H\sum_{k=1}^{K}(\mathbf{w}_k\mathbf{w}_k^H)\mathbf{R}\mathbf{d}^m. \end{aligned} \tag{27}$$

The advantages of the proposed eigenvector combining is clear, that is, it requires only few training OFDM symbols for array weight calculation, and can utilize almost all part of received signal power. When the SMI algorithm is used for array weight calculation, the computational complexity is still $O(N_p^3 \times (L_g + L_u)^3)$.

VI. Relationship between Eigenvector combining and Post-FFT Type

The FFT operation is linear, so multiplication of a weight at the FFT input is equal to that of the same weight at all the FFT outputs. Taking into consideration this FFT input/output relationship, the structure of the proposed eigenvector combining shown in Fig.4 (a) can be changed into that of the very post-FFT type shown in Fig.4 (b). Therefore, *the eigenvector combining is a realization of a post-FFT array antenna through a pre-FFT signal processing.*

The equivalence between them for the case of no interference can be proved as follows. When there is no interference, the generalized eigenvalue problem given by Eq.(20) becomes just an eigenvalue problem, where the eigenvectors are mutually orthogonal. Therefore, taking the noise subspace into consideration, the summation term up to N in Eq.(27) is simplified into

$$\sum_{k=1}^{K}(\mathbf{w}_k\mathbf{w}_k^H) = \mathbf{I}_N, \tag{28}$$

so Eq.(27) becomes

$$v^m = (\mathbf{H}'\mathbf{d}^m)^H (\mathbf{R}\mathbf{d}^m). \tag{29}$$

Equation (29) shows the equivalence between the eigenvector combining and the post-FFT type, *where the m-th final subcarrier output is the sum of complex envelope-weighted m-th FFT outputs.*

VII. Numerical Results and Discussions

For computer simulation, an eight-element circular array with *half*-wavelength adjacent spacing is employed. A coherent quadrature phase shift keying (QPSK) scheme is assumed with a half-rate convolutional encoding/Viterbi decoding with a constraint length (K) of 7 and a (12×8) block interleaver. One OFDM symbol is composed of 80 samples, where the guard interval length is 16 samples and the useful symbol length is 64 samples. Here, the OFDM symbol is generated with the 64-point inverse FFT (IFFT), where only 48 subcarriers convey information, 4 subcarriers are known pilot signals and the other 12 subcarriers are virtual subcarriers. The one signal burst is composed of 2 OFDM symbol-long preamble and 10 OFDM symbol-long payload. In the preamble, one training OFDM symbol is used for estimation of channel impulse response and the other training OFDM symbol estimates the correlation matrix of the received signal.

We restrict our attention to an uplink transmission, where the position of a base station (BS) is sufficiently high so that few local scatterings occur. Therefore, spatial fading at the BS will be correlated with an exact correlation depending on the BS antenna element spacing, the carrier frequency used, and the angle spread observed at the BS array[10]. Figure 5 shows a spatio-tempral channel model, where there are three desired signals arriving within the guard interval and as a cochannel interfering signal, there is one delayed signal beyond the guard interval. Each signal arrives forming a cluster with an angle spread, which is composed of eight waves, the envelope is Rayleigh-distributed with the same power and the DoA is uniformly distributed in [0 deg., 360 deg.). The arrival time of the desired signal is uniformly distributed within the guard interval, whereas that of the interfering signal is uniformly distributed within the useful symbol interval beyond

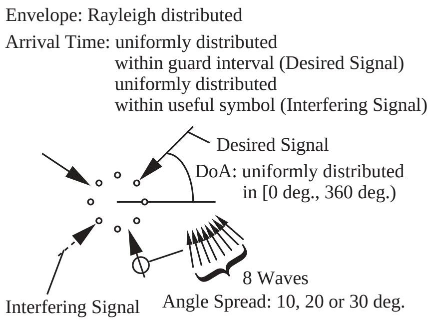


Fig. 5. A spatio-temporal channel model.

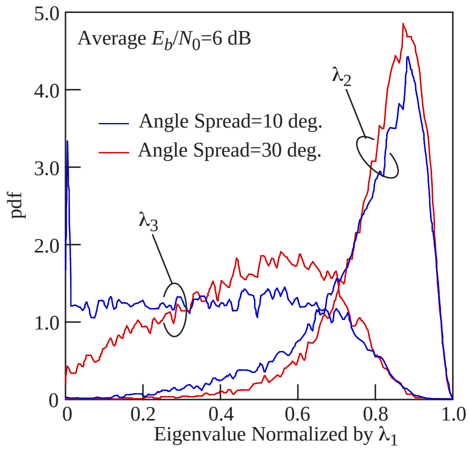


Fig. 6. Eigenvalue distribution.

the guard interval. Furthermore, the channel fading is slow enough so that the channel impulse response does not significantly change over one signal burst.

Figure 6 shows the simulated probability density function (pdf) of eigenvalues λ_2 and λ_3 normalized by the first largest eigenvalue λ_1, where the ratio of the average received energy per bit to the white noise power spectral density per antenna (E_b/N_0) is 6 dB and 20000 outcomes are employed. The occurrence of large λ_k substantially increases when the angle spread increases (here, from 10 deg. to 30 deg.). This suggests us that the BER performance of the eigenvector combining can become better as the angle spread increases.

Figure 7 shows the BER versus the average E_b/N_0 of the proposed eigenvector combining. Here, a threshold parameter λ_{th} is newly defined, where the eigenvectors associated with $\lambda_k \geq \lambda_{th} \cdot \lambda_1$ are employed for combining. This means that the eigenvector combining with $\lambda_{th} = 1$ is equivalent to the conventional pre-FFT array antenna. The figure clearly shows that the BER becomes better when λ_{th} decreases and the angle spread increases (in the other words, the fading correlation among antenna elements decreases), as expected intuitively.

Figure 8 shows the BER versus λ_{th} of the proposed scheme. The BER performance is improved as the λ_{th} decreases, i.e., more eigenvectors are employed for combining. No threshold effect is observed which is typical in subspace approaches such as MUSIC.

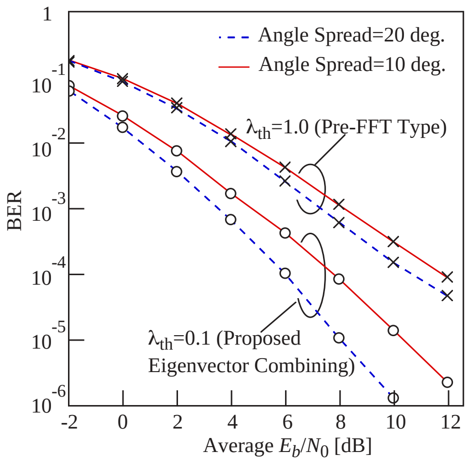


Fig. 7. BER performance.

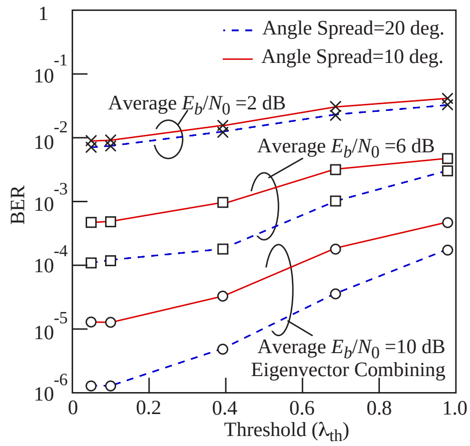


Fig. 8. BER performance versus λ_{th}.

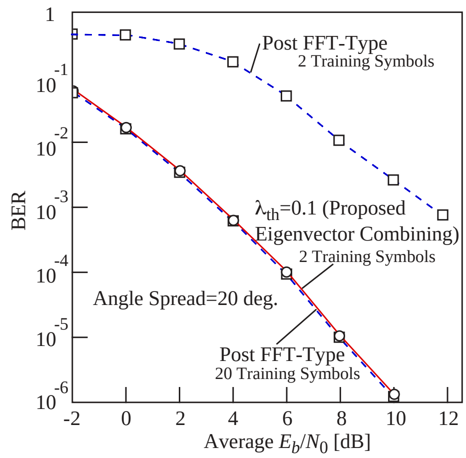


Fig. 9. BER comparison between eigenvector combining and post-FFT type.

Finally, Figure 9 shows the BER comparison between the eigenvector combining and the post-FFT array antenna. *When there is interference*, it has not been completed to theoretically prove the equivalence between the two schemes, but the figure clearly shows that the performance of the eigenvector combining with just two training symbols is almost the same as that of the post-FFT array antenna with twenty training symbols. According to our calculation, the achievable *SINR* of the eigenvector combining is smaller than that of the post-FFT array antenna by less than 1 %.

VIII. Conclusions

This paper proposed a novel pre-FFT type OFDM adaptive array antenna called “Eigenvector Combining.” The eigenvector combining is a realization of a post-FFT type OFDM adaptive array antenna through a pre-FFT signal processing, and has the advantageous characteristics of the two types of array antennas: excellent performance due to power efficiency inherent in post-FFT type and less computational complexity and shorter training symbols due to pre-FFT signal processing proper to pre-FFT type.

Numerical results showed that, for the case of three desired signals and one co-channel interfering signal, with two training symbols, the post-FFT array antenna cannot work well at all, on the other hand, the proposed eigenvector combining shows excellent BER performance close to the lower bound.

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