Spreading Sequence Assignment in the Downlink of OFCDM Systems Using Multiple Transmit Antennas

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Abstract— **In communication systems based on code-division multiple access, a proper assignment of spreading sequences can help reducing the multiple access interference. In this paper, an optimized assignment of spreading sequences is proposed for down-link Orthogonal Frequency and Code Division Multiplexing (OFCDM) systems applying transmit beamforming at the base station. Assignment involves a selection of the needed subset of spreading sequences and a distribution of these selected sequences according to the spatial signature of mobile terminals. A general assignment rule is exposed for any kind of sequences, which requires some computational effort. A very simple assignment rule is then proposed in the particular case of Walsh-Hadamard sequences. Simulation results over realistic transmission scenarios emphasize the necessity to optimize the assignment of spreading sequences as well as the efficiency of the proposed solution whatever the system load and the beamforming efficiency.**

I. INTRODUCTION

Over the past few years, the Orthogonal Frequency and Code Division Multiplexing (OFCDM) transmission scheme, also referred to as Multi-Carrier Code Division Multiple Access (MC-CDMA), has been widely studied by the research community for its capability to match the requirements of a 4G mobile cellular system 0-[2]. OFCDM combines the Orthogonal Frequency Division Multiplex (OFDM) and CDMA schemes by spreading in the frequency domain. Thus, it offers robustness against multipath propagation and flexibility of resource allocation by assigning to the different users one or several spreading sequences (multi-code) of same or different lengths (variable spreading factor). To ensure orthogonality among the signals of different users, Walsh-Hadamard (W-H) spreading sequences are often considered.

However, as a CDMA-based scheme, OFCDM suffers from Multiple Access Interference (MAI) arising from propagation of the signal through the multipath channel. Mainly two different strategies have been investigated to mitigate MAI: Improved detections at the receiver side, e.g. multi-user detection, and pre-filtering techniques at the transmitter side. For the Down-Link (DL), i.e. from the Base Station (BS) to Mobile Terminals (MT), algorithms of low complexity and power consumption are needed at the MT side, whereas an increase of complexity is tolerable at the BS side.

Spatial pre-filtering, i.e. transmit Beam-Forming (BF), at the BS is an alternative way to mitigate MAI in the DL without any increase of the MT complexity [4]. This strategy benefits form the spatial separation of MTs of the same cell to form beams towards the propagation directions of the desired MT, which limits the interference towards other MTs. DL transmit BF assumes that the directions of departure (DOD) of users' signals, i.e. the spatial signatures of users, are known at the BS prior to transmission. For instance, spatial knowledge may be extracted from channel estimation in the Up-Link (UL).

An additional mean to reduce MAI in the DL consists in optimizing the assignment of spreading sequences required by the different active MTs. In [5], we proposed such an optimization for DL MC-CDMA systems with a single transmit antenna. We showed that for non-full load system, given a set of available spreading sequence, e.g. W-H, the system performance considerably depends on the subset of sequences chosen to satisfy the needs of all MTs. Therefore, the proposed criterion and associated methodology optimize the spreading sequence selection. The selection does not rely on any specific assumption but only some correlation between sub-channels over which the data symbols are spread. This approach was confirmed in [6] where another assignment criterion is proposed. In both methods, there remains a degree of freedom to distribute the sequences of the selected subset among the different MTs.

In this paper, we present a strategy for spreading sequence assignment in order to reduce MAI in DL OFCDM systems using transmit BF [7] at BS. The assignment scheme consists not only in the selection of an optimized subset of sequences as in [5] but also in an efficient distribution of these selected sequences among the different active MTs. The selection of the subset is optimized in such a way that it is independent on the BF algorithm and the number of transmit antennas. The distribution of the selected sequences is only based on the spatial signatures of MTs and results in a performance improvement of the system for any system load.

The remainder of the paper is organized as follows. In section 2, we present the proposed DL OFCDM system model using transmit BF at BS. We introduce the impact of the channel correlation in time and frequency on the assignment of spreading sequences. In section 3, we present a general method for spreading sequence assignment that benefits from the channel correlation and is applicable to any spreading sequence family. We detail a very simple assignment scheme in the case of Walsh-Hadamard sequences. In section 4, we validate the proposed spreading sequence assignment in a realistic 4G mobile cellular environment. Finally section 5 gives concluding remarks and prospects.

II. SYSTEM DESCRIPTION

The proposed system with M transmit antennas is depicted in Fig. 1. At the BS side, for each MT_k (k=1…K), binary information is encoded and interleaved. Then, data symbol d_k, e.g. generated from a Quaternary Phase Shift Keying (QPSK) alphabet, is filtered with BF vector $\mathbf{w}_k$ of size M according to the spatial information of MTs' signals. Each of the M outputted symbols is spread on L chips using the sequence vector $\mathbf{c}_k$. For clarity, we assume that each MT uses only one sequence of same length L. These sequences compose the active set of sequences $\Omega(K)=\{\mathbf{c}_1,\ldots,\mathbf{c}_K\}$. This set is determined by the BS on the basis of the spatial information of MTs, which is also used for BF. For instance, this information may be the average DOD (ADOD) of MT's signals. In practice, the active set of spreading sequences is updated regularly, e.g. at each frame, and communicated to the MTs through a control channel. The LM chips of all MTs are summed and demultiplexed in order to be transmitted on the appropriate antenna. On each antenna branch, chip mapping puts the L demultiplexed components onto L sub-channels of the OFDM frame. A frame contains $N_F N_T$ different sub-channels, where N_F is the number of sub-carriers and N_T is the number of OFDM symbols. This OFCDM system thus allows the transmission of $N_F N_T/L$ data symbols per user in one frame. Finally, on each antenna branch, OFDM modulation is carried out and a guard interval Δ is inserted to absorb the multipath spread of the channel.

Thanks to the guard interval and assuming that the coherence time of the channel is much longer than the duration of the OFDM symbol, the propagation channel for MT_i can be represented by a flat fading coefficient on each sub-channel. Then, vector $\mathbf{h}_i$ of size ML gathers the fading coefficients for each antenna and each sub-channel for MT_i.

From the signal received by a single antenna at MT_i, the guard interval is removed and OFDM demodulation is carried out. From the $N_F N_T$ sub-channels of the frame, chip de-mapping gathers the signal received on the L sub-channels dedicated to the transmission of data symbol d_i. Since the channel is selective in frequency due to multipath and in time due to Doppler variations, the orthogonality among users' signals may be corrupted. Assuming that a Single-User Detection (SUD) scheme is employed to limit the MT complexity, equalization vector $\mathbf{g}_i$ of size L compensates for the channel selectivity. Then, despreading with the appropriate sequence $\mathbf{c}_i$ yields an estimate $\hat{d}_i$ of transmitted data symbol d_i as:

$$\hat{d}_i = \underbrace{\tilde{\mathbf{c}}_i^H \cdot \left(\tilde{\mathbf{g}}_i \circ \mathbf{h}_i \circ \tilde{\mathbf{w}}_i^* \circ \tilde{\mathbf{c}}_i\right) d_i}_{\text{Desired Signal}} + \underbrace{\tilde{\mathbf{c}}_i^H \cdot \left(\tilde{\mathbf{g}}_i \circ \mathbf{h}_i \circ \sum_{k=1,k\neq i}^{K} \left(\tilde{\mathbf{w}}_k^* \circ \tilde{\mathbf{c}}_k\right) d_k\right)}_{\text{MAI}} + \underbrace{\mathbf{c}_i^H \cdot \left(\mathbf{g}_i \circ \mathbf{n}_i\right)}_{\text{Noise}} \qquad (1)$$

where * defines the complex conjugate operator, ^{H} defines the conjugate transpose operator and $\circ$ defines the vector multiplication element by element. To represent the recombination of signals at the single receive antenna, we define an extended vector notation indicated by ~. Then, for each MT_k, $\tilde{\mathbf{c}}_k$ is a spreading vector of size ML repeating M times vector $\mathbf{c}_k$, i.e. $\tilde{\mathbf{c}}_k^T = (\mathbf{c}_k^T,\ldots,\mathbf{c}_k^T)$ since the same spreading is applied on all transmit antenna branches. Similarly, $\tilde{\mathbf{g}}_i$ is an equalization vector of size ML repeating M times vector $\mathbf{g}_i$, i.e. $\tilde{\mathbf{g}}_i^T = (\mathbf{g}_i^T,\ldots,\mathbf{g}_i^T)$ since the same equalization vector is applied to the signals coming from the different transmit antennas. $\tilde{\mathbf{w}}_k$ is a BF vector of size ML repeating L times vector $\mathbf{w}_k$, the ℓ–th component of $\tilde{\mathbf{w}}_k$ being defined as:

$$\tilde{\mathbf{w}}_k(\ell) = \mathbf{w}_k(\lfloor \ell / L \rfloor) \quad \forall \ell = 0,\ldots,LM-1 \qquad (2)$$

where $\mathbf{w}_k(\ell)$ is ℓ–th component of $\mathbf{w}_k$ and $\lfloor x \rfloor$ is the floor of x. $\mathbf{n}_i$ is an additive white gaussian noise vector of size$L$.

Finally, soft de-mapping, channel de-interleaving and channel decoding yields the binary information stream.

III. SEQUENCE ASSIGNMENT USING SPATIAL INFORMATION

Let Ω be a set of L sequences, e.g. W-H, and $\Omega(K)$ a subset of K sequences taken out from Ω ($K<L$) to satisfy the needs of K active MTs. Sequence assignment has to be carried out dynamically by the BS since the number K of requested spreading sequences may change in the network, e.g. at each new frame.

A. Optimum Sequence Assignment for Single Antenna System

For OFCDM systems using a single transmit antenna, we already demonstrated in [5] that a correlation of the L sub-channels involved in the chip mapping allows us to optimize the selection of $\Omega(K)$ among Ω. In that case, a given MT is subject to the interference of all other MTs. Therefore, the

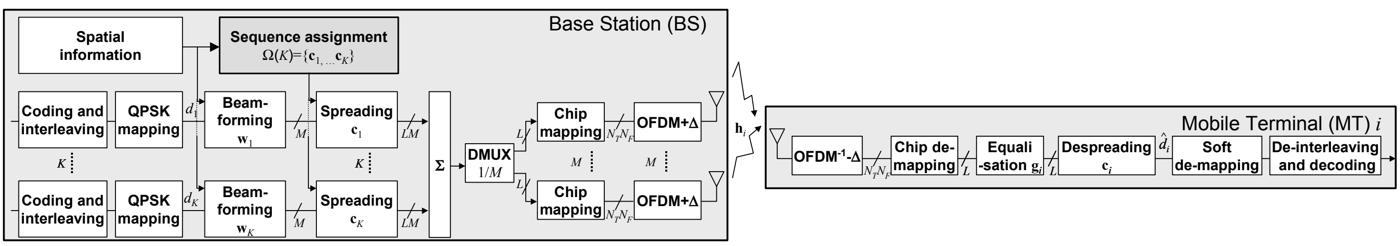

Fig.1: Block diagram of a down-link OFCDM system with M transmit antennas at the BS.

subset $\Omega(K)$ has to be optimized as a whole and the way the sequences are ordered, i.e. distributed among the different MTs, has no impact.

An optimization of the required subset based on an exhaustive search involves the analysis of $L!/[(L-K)!K!]$ different candidate subsets. In [5], we introduced a simplified cost function $J(\Omega(K))$ that is related to the level of interference caused by the sequences of $\Omega(K)$ and allows us to select an optimized subset of sequences without knowledge of the propagation channels. It only assumes a correlation of fading on sub-channels over which data symbols are spread, which decreases with respect to the distance between these sub-channels. For clarity, we rewrite $J(\Omega(K))$ as:

$$J(\Omega(K)) = \min_{\{\mathbf{c}_i,\mathbf{c}_j\} \in \Omega(K)^2, i \neq j} T\left(\mathbf{x}^{(i,j)}\right)$$
$$\text{with} \quad T\left(\mathbf{x}^{(i,j)}\right) = \sum_{\ell=1}^{L-1} \left| \operatorname{sgn}\left(x^{(i,j)}(\ell+1)\right) - \operatorname{sgn}\left(x^{(i,j)}(\ell)\right) \right| \tag{3}$$

where $x^{(i,j)}(\ell)$ is the ℓ-th component of vector $\mathbf{x}^{(i,j)}=\mathbf{c}_i^* \circ \mathbf{c}_j$. $T(\mathbf{x}^{(i,j)})$ is the number of sign changes between consecutive components of $\mathbf{x}^{(i,j)}$. This is a measure of how a joint use of $\mathbf{c}_i$ and $\mathbf{c}_j$ will impact MAI: A good (resp. bad) pair of sequences $\mathbf{c}_i$ and $\mathbf{c}_j$ is such that $T(\mathbf{x}^{(i,j)})$ is maximum (resp. minimum).

A maximization of $J(\Omega(K))$ over any candidate subset yields an optimum subset $\Omega_0(K)$ defined as:

$$\Omega_0(K) = \arg \max_{\Omega(K) \subset \Omega} J(\Omega(K)) \tag{4}$$

Applying (3) and (4) aims at reducing the maximum interference caused by any pair of sequences belonging to any subset $\Omega(K)$. It has to be noted that this optimization process can be carried out off-line, once for all, for any value of K and stored in Look-Up Tables (LUTs).

B. Optimum Sequence Assignment for Multi-Antenna Systems

For OFCDM systems using multiple transmit antennas, the impact of correlation on the choice of spreading sequences remains similar. However, a given MT_i is only subject to the interference of MTs that cannot be separated in space from MT_i with transmit BF. So, a subset of sequences optimized for MT_i may be sub-optimum for MT_j ($j \neq i$) and the way the spreading sequences are ordered in $\Omega(K)$, i.e. distributed among MTs, impacts the performance. Here, an optimization of the required subset based on exhaustive search would involve the analysis of $L!/[(L-K)!]$ different candidates, i.e. 10^{35} subsets for L=K=32. Moreover, the subset of sequences should be optimized on-line and periodically taking into account the variations of the spatial signatures of all MTs. Since this requires a prohibitive processing power, such an approach is not realistic. To simplify the problem, we propose an optimization in three steps as depicted in Fig. 2.

<u>*Step 1*</u>: We first assume that despite transmit BF all assigned spreading sequences potentially interfere so that we select a subset $\Omega_0(K)$ of spreading sequences as in a single antenna scenario according to (3) and (4). So, the exhaustive search only involves $L!/[(L-K)!K!]$ subsets for any value of K and the optimum subsets are available in LUTs. Thus, for $K<L$, we avoid to assign sequences being responsible for a large part of MAI in a single antenna system, which also reduces MAI in a multi-antenna system.

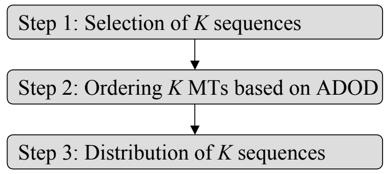

Fig. 2: General optimization process of sequence assignment.

<u>*Step 2*</u>: The BS orders active MTs by their ADOD so that MT_1 has the smallest ADOD and MT_K the largest one. Hereby, we assume that the difference of ADODs is representative of the spatial separation of MTs' signals obtained by transmit BF.

<u>*Step 3*</u>: From the selected subset $\Omega_0(K)$, the BS distributes, i.e. orders, the sequences so that interference among MTs having similar spatial signatures is minimized. For that purpose, given any ordered subset $\overline{\Omega}(K)=\{\mathbf{c}_1,\ldots,\mathbf{c}_K\}$, the distribution of sequences must be jointly optimized for sequences $\mathbf{c}_i$ and $\mathbf{c}_j$ only if the difference between i and j is low, i.e. if MT_i and MT_j have close ADODs. Thus, by introducing P subgroups $\overline{\Omega}^p(K)$ of sequences that are likely to interfere, an optimization process similar to (4) yields the optimum ordered subset $\overline{\Omega}_0(K)$ as:

$$\overline{\Omega}_0(K) = \arg \max_{\overline{\Omega}(K) \subset \Omega_0(K)} \ \min_{\overline{\Omega}^p(K) \subset \overline{\Omega}(K) \, p=1..P} J\left(\overline{\Omega}^p(K)\right) \tag{5}$$

Expression (5) aims at minimizing the maximum of interference occurring within each subgroup of sequences. The definition of these subgroups highly depends on the BF scheme. For instance, if orthogonal fixed beams are used, the subgroups gather the sequences assigned to the MTs using the same beam. In case of adaptive BF, one subgroup may be defined for each MT including the sequences assigned to neighboring MTs. Besides, it has to be noted that the exhaustive search proposed in (5) involves $K!$ different ordered subsets $\overline{\Omega}(K)$, which may still be a limitation in practice for systems with high loads.

C. Application to Walsh-Hadamard Sequences

When applied to W-H sequences for any value of $K \leq L$, the computational complexity involved by the optimization process described in Fig.2 is drastically reduced.

As the first step, applying (3) and (4) leads to an optimum subset $\Omega_0(K)$ composed of the K first lines of the W-H matrix $\mathbf{C}_L$ of size $L \times L$ defined as:

$$\mathbf{C}_L = \begin{bmatrix} \mathbf{C}_{L/2} & \mathbf{C}_{L/2} \\ \mathbf{C}_{L/2} & -\mathbf{C}_{L/2} \end{bmatrix} \qquad \text{with } \mathbf{C}_1 = [+1] \tag{6}$$

In other words, in a single antenna system, an optimum assignment does not require any computational effort. Only periodic signaling is needed to ensure that assignment is still optimum each time a new MT becomes active or inactive.

In the second step, MTs are sorted based on their ADOD. The complexity involved by this step is very limited since DODs may have been already extracted for BF.

For the final step, we consider the natural order of W-H sequences taken out from (6), i.e. $\mathbf{c}_1$ is the first line of $\mathbf{C}_L$, $\mathbf{c}_2$ the second line etc. This order has the following properties:

$$\begin{aligned} T\left(\mathbf{x}^{(i,i+1)}\right) &= L-1 \quad \forall i \text{ odd} \\ T\left(\mathbf{x}^{(i,i+1)}\right) &\geq L/2 \quad \forall i \text{ even} \end{aligned} \tag{7}$$

where $T(\mathbf{x}^{(i,j)})$ is the approximation defined in (3).

From (7), each sequence $\mathbf{c}_i$ of $\mathbf{C}_L$ has one neighboring sequence that may induce a minimum interference with $\mathbf{c}_i$ compared to any other sequence of $\mathbf{C}_L$. For $2 \leq i \leq L-1$, the other neighboring sequence of $\mathbf{c}_i$ may induce an interference level below the average level $L/2$ occurring between any pair of sequences. Therefore, we choose to distribute the W-H sequences in the natural order to the different ordered MTs, which ensures that each MT has neighboring MTs with low interference. This very simple assignment scheme is summarized in Fig. 3.

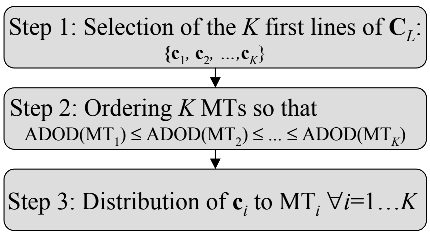

Fig. 3: Simple assignment process in the case of W-H sequences.

IV. Numerical Results

For evaluating the performance of the proposed assignment strategy, we consider an outdoor scenario, whose parameters are derived from the European IST Matrice project [3] and summarized in Table 1.

TABLE I. SIMULATION PARAMETERS

Carrier frequency	5 GHz
Sampling frequency	57.6 MHz
FFT size	1024
Number of modulated carriers	736
Slot duration (in OFDM symbols)	30
Cyclic prefix size (in samples)	216
Multi-path channel model	BRAN channel E
Spreading codes	Walsh-Hadamard
Spreading factor (L)	32
Modulation alphabet	QPSK
Number of antennas (M)	1, 4, 8
Blocksize of information bit	920
Convolutional coding scheme	R_C=2/3, κ=7

The time channel model, which is referred to as BRAN channel E [8], is extended to a space-time model by allocating a DOD to each of its paths. For each user, the DODs are randomly chosen within an angle spread of 30° around the main DOD, which itself is randomly chosen in a 120° sector for each user. The BS is equipped with a half wavelength-spaced uniform linear array. The mapping of chips is performed on adjacent sub-carriers of the same OFDM symbol. At the transmitter, Eigen-BF, which has been proposed in [9] and applied to MC-CDMA in [7], is considered with a perfect knowledge of spatial information. Eigen-BF determines the antenna coefficients from an Eigenvalue decomposition of the channels spatial covariance matrix. The Eigenvector corresponding to the maximum Eigenvalue is used as BF vector. At the receiver side, we consider Equal Gain Combining, with perfect channel estimation. Note that applying an interference-based SUD such as Minimum Mean Square Error Combining (MMSEC) [1] is not straightforward in this case since the amount of remaining interference after BF cannot be precisely known at the MT side. When channel coding is employed, convolutional coding (rate R_C=2/3, constraint length κ=7) and random bit interleaving are considered.

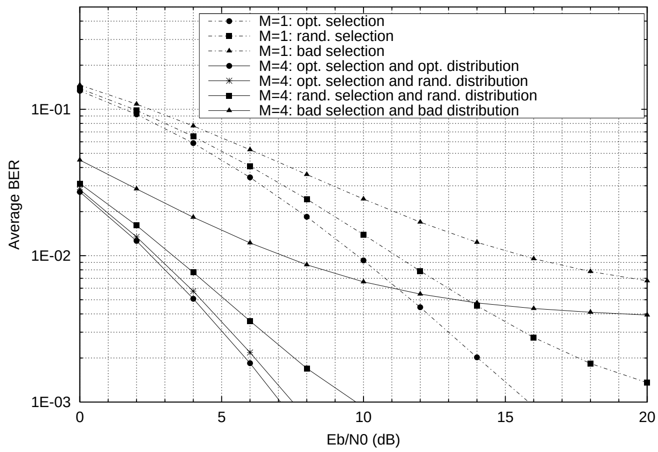

Fig.4: Performance of assignment without coding (K=16).

In Fig. 4, the Bit Error Rate (BER) performance of different assignment schemes is depicted according to the ratio of transmit bit energy Eb over noise spectral density N0. Single (M=1) and multiple antenna (M=4) systems are considered with K=16 active users corresponding to a half load. For M=1, a distribution according to the spatial information has no impact and the assignment schemes only differ by the way they select the subset of sequences. To achieve BER=10^{-2}, an optimum selection of sequences provides a gain of 1.5 dB compared to a random selection and a gain of 6 dB compared to a bad selection. For M=4, even if BF already reduces the amount of MAI at each MT, an optimization of spreading sequence assignment further reduces MAI. Compared to a bad selection and distribution of sequences, an optimum assignment achieves a gain of 4.5 dB. A slight loss is experienced when the sequences are well selected but randomly distributed, which emphasizes the gain of distributing the sequences according to the ADOD of users' signals. Finally, a random selection and distribution lead to a 1 dB loss, which underlines the necessity to select the sequences properly.

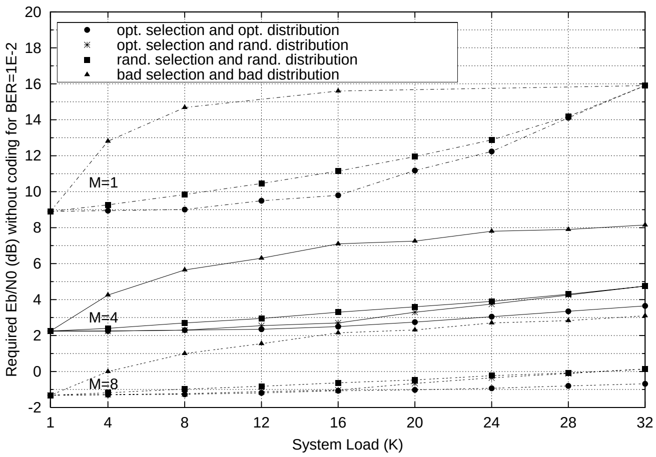

Fig.5. Influence of system load.

In Fig. 5, we represent the influence of the system load on the required Eb/N0 to achieve BER=10^{-2} without coding for different assignment schemes and different numbers of antennas. For a single user system, the performance does not depend on the selected sequence. For *M*=1, the gain provided by the optimum scheme increases up to 6 dB for half-load and decreases regularly down to 0 dB for full load. Indeed, with *K*=*L*, there are no more degrees of freedom for an optimized selection since all sequences must be allocated. Moreover, with *M*=1, MTs' signals cannot be separated in space so that the distribution of sequences has no impact on the performance. In contrast, for a larger number of antennas, even if the selection of sequences has no impact at full load, an optimum distribution of them according to the ADOD of MTs provides a significant performance improvement. For instance, with 4 antennas, the gain provided by an optimized selection and distribution of sequences increases up to 4.6 dB compared to a bad assignment. It has to be noted that such a bad assignment may statistically occur in the system if sequence assignment is not properly handled. Compared to a random assignment, the average gain of the proposed assignment grows with the system load up to more than 1 dB.

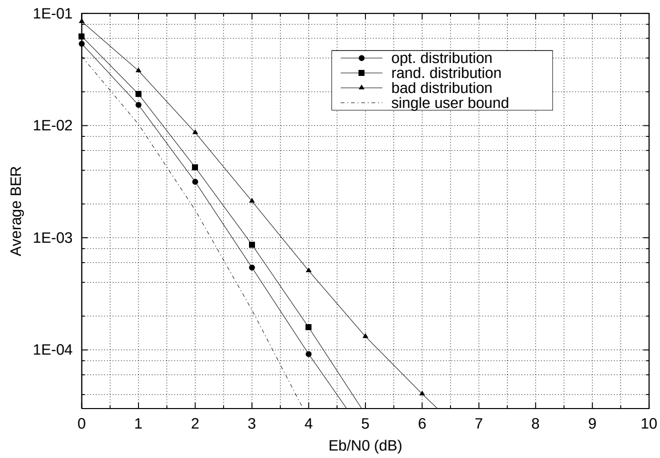

Fig.6. Performance of assignment with coding (*M*=4, *K*=32).

With 8 antennas, there is a slight reduction of the performance improvement brought by the proposed assignment since the better BF efficiency reduces the impact of sequence assignment on the residual MAI.

In Fig. 6, the performance of the different assignment strategies is compared taking into account channel coding. A fully loaded system (*K*=32) is considered with *M*=4 transmit antennas. Here, the different assignment schemes only differ by the way they distribute the full set of W-H sequences among the active MTs. The good performance of the optimized assignment scheme is confirmed with a BER=10^{-4} achieved for Eb/N0=3.9 dB. This is only a 0.6 dB loss compared to the performance for a single user.

V. Conclusion

When transmit beamforming is employed in an OFCDM downlink transmission, the BS can assign the spreading sequences required by active MTs according to the spatial signature of their signals. We showed in this paper that such an optimization can lead to performance improvement whatever the system load ($K \leq L$) and the beamforming efficiency. With W-H sequences, this optimized assignment induces no specific complexity increase: spatial signatures are already available for beamforming and signaling is required anyway in order to inform each MT about the sequence to use for de-spreading. Finally, the important performance gain already achieved by the proposed method is expected to increase for higher loads ($K>L$) if the system allows sequence re-assignment thanks to beamforming.

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References

[1] S. Hara, R. Prasad, 'Overview of multicarrier CDMA,' *IEEE Communications Magazine*, vol. 35, pp. 126-133, Dec. 1997.
[2] H. Atarashi, N. Maeda, S. Abeta, M. Sawahashi, 'Broadband Wireless Access based on VSF-OFCDM and MC/DS-CDMA,' *Proc. of IEEE PIMRC 2002,* vol. 3, pp. 992-997, Sept. 2002.
[3] IST MATRICE project, web site http://www.ist-matrice.org.
[4] A.F. Naguib, A.J. Paulraj, T. Kailath, 'Capacity improvement with base-station antenna arrays in cellular CDMA,' *IEEE Trans. on Vehic. Tech.*, vol. 43, no. 3, pp. 691-698, Aug. 1994.
[5] D. Mottier, D. Castelain, "A Spreading Sequence Allocation Procedure for MC-CDMA Transmission Systems," *Proc. of IEEE VTC 2000 Fall,* vol. 3, pp. 1270-1275, Sept. 2000.
[6] Q. Shi, M. Latva-aho, "Simple Spreading Code Allocation Scheme for DL MC-CDMA," *Electronics Letters*, vol. 38, no. 15, pp. 807-809, Jul. 2002.
[7] T. Sälzer, D. Mottier, "Downlink Strategies Using Antenna Arrays for Interference Mitigation in Multi-Carrier CDMA," *Proc. of Fourth Workshop on Multi-Carrier Spread Spectrum (MC-SS) and related topics*, Sept. 2003.
[8] J. Medbo, 'Channel Models for Hiperlan/2 in different Indoor Scenarios,' *ETSI BRAN doc.* 3ERI085b, 1998.
[9] S. A. Jafar, S. Vishwanath, and A. Goldsmith, "Channel capacity and beamforming for multiple transmit and receive antennas with covariance feedback," *Proc. of IEEE ICC 2001*, vol. 7, pp. 2266-2270, June 2001.