

Impact of the Spreading Sequences on the Performance of Forward Link MC-CDMA Systems

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Abstract—In this paper, we investigate the impact of the spreading sequences on the performance of forward link MC-CDMA systems. We show that between the extreme cases of uncorrelated and highly correlated channels, the spreading sequences have a great influence on the mutual interference power. This is true for Walsh-Hadamard and Fourier orthogonal sequences where the aperiodic correlation function varies drastically with respect to the spreading sequences. Thus, in this context, selecting the subset of less mutually interfering sequences for a given system load can significantly improve the system performance. Based on this observation, we propose a recursive algorithm that at a given recursion selects the sequence which has the less mutual interference powers with the already selected sequences. This algorithm is shown to provide significant gains up to 3.5 dB in terms of minimum achieved SINR when comparing to bad sequence selection. This algorithm requires knowledge of the equalized channel correlations for all active users, which may be difficult to provide in practice. To overcome this difficulty, we show that by making some assumptions on the channel correlation model, one can greatly reduce the algorithm knowledge requirements without significant loss in performance.

I. INTRODUCTION

Multi-Carrier (MC) transmission techniques that combine Orthogonal Frequency Division Multiplexing (OFDM) with Code Division Multiple Access (CDMA) are considered as potential candidates for the forward link air interface of 4G wireless communication systems. In particular, MC-CDMA schemes seem to fulfill quite well the 4G forward link air interface requirements. In the literature [1]-[6], MC-CDMA refers to a well-known OFDM-CDMA combination that performs spreading along the time and frequency dimensions, i.e. each CDMA chip is transmitted on one assigned sub-carrier during the time interval of one assigned OFDM symbol. Two different mappings of the CDMA chips are generally envisaged. The first mapping aims at placing the CDMA chips on independently faded sub-carriers in order to make full use of the diversity effect [1]-[4]. However, this breaks the orthogonality between the spreading sequences and increases the level of multiple access interference (MAI). In contrast to the first mapping, the second mapping aims at placing the CDMA chips on highly correlated faded sub-carriers in order to preserve orthogonality among the spreading sequences [5][6]. However, this mapping cannot achieve any diversity gain, which is left to channel coding and bit interleaving.

In this study, we investigate the influence of the spreading sequences on the MAI power for forward link MC-CDMA

systems in the realistic case of correlated faded sub-carriers. As it will be shown later, between the extreme cases of uncorrelated and highly correlated channel coefficients, the choice of the spreading sequences for a given system load can have a great influence on the MAI power, and consequently on the system performance. This is particularly true for Walsh-Hadamard and Fourier orthogonal sequences [10], where the aperiodic correlation function varies drastically with respect to the spreading sequences. Thus, in such a case, selecting the subset of the less mutually interfering sequences can significantly improve the system performance. From this observation, we investigate the optimal selection of the spreading sequences and propose a recursive algorithm. This algorithm proceeds recursively in the sense that at a given recursion, it selects the sequence which has the less mutual interference powers with the already selected sequences for a given system load. The gain in performance achieved by this algorithm as well as its implementation costs are discussed in details in the sequel.

The rest of this paper is organized as follows. Section II describes the forward link MC-CDMA system model. In Section III, we discuss the influence of the spreading sequences on the MAI power and present the recursive algorithm for optimal spreading sequence selection. In Section IV, computer simulations are carried out to validate the theoretical analysis done in Section III. Finally, our conclusions are drawn in Section V.

II. SYSTEM DESCRIPTION

We consider the forward link transmission to K users. The transmission occurs simultaneously and synchronously using OFDM modulation with N_c available sub-carriers.

A. MC-CDMA Transmitter Model

A block diagram of the baseband model of the MC-CDMA transmitter in the forward link is depicted in Figure 1. After channel encoding and interleaving, the binary information of user k is mapped to QPSK modulation symbols. The resulting symbol stream $\{d_k\}$ is then Serial/Parallel converted into P parallel streams. Then, each parallel stream is spread using the spreading sequence $\{c_k\}$ assigned to user k . Component-wise summation is then performed over the resulting chip streams of the K active users. The multi-user chips are then mapped to the time-frequency bins according to the chip mapping method. Two chip mapping methods are generally considered.

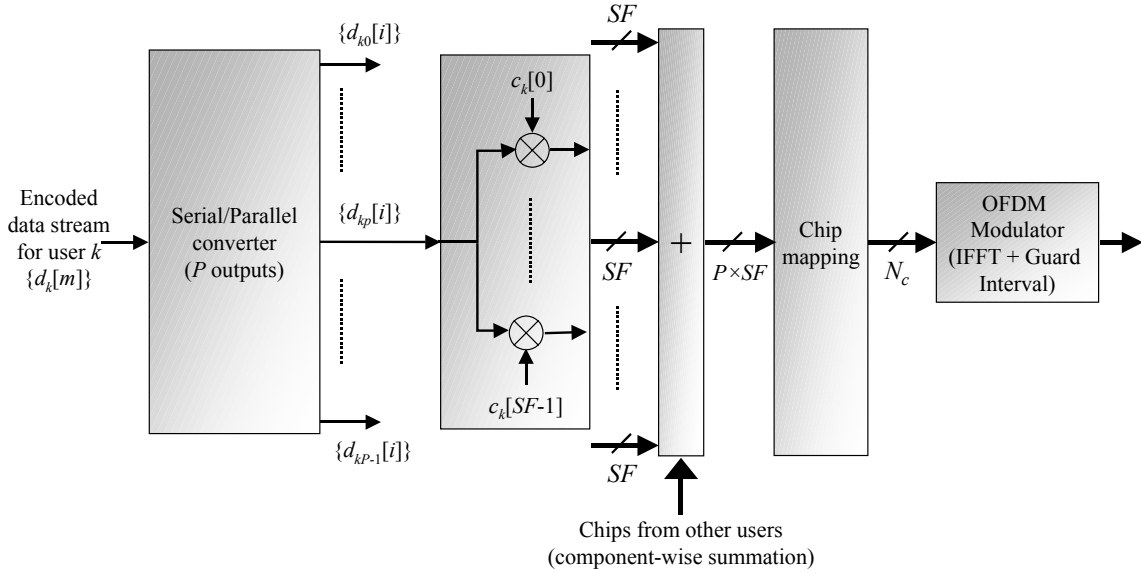


Figure 1: Block diagram of the MC-CDMA baseband transmitter in the forward link.

In the first method, the CDMA chips of the same data symbol are mapped to the time-frequency bins over which the lowest multi-path fades correlation can be achieved. The aim here is to benefit from the time-frequency diversity gain. In contrast to the first method, the second method aims at preserving orthogonality among the multi-user signals by mapping the chips to the time-frequency bins over which the highest multi-path fades correlation can be achieved. For more details of the chip mapping methods, the lecturer is referred to [6]. After the chip-mapping operation, the multi-user chips are sent to the OFDM modulator, which performs the inverse fast Fourier transform (IFFT) operation and the guard interval insertion. The baseband signal is then RF modulated and transmitted through the multi-path channels of the K active users.

B. Multi-path Channel Model

As assumed in [3], we consider a normalized wide-sense stationary uncorrelated scattering (WSSUS) channel, with maximum delay smaller than the guard interval duration, resulting in zero inter-symbol interference. Furthermore, the channel is assumed to be time-invariant over the useful OFDM symbol duration T_u , and therefore the channel effect on sub-carrier n at the time interval $[iT_s, (i+1)T_s[$ of the i -th OFDM symbol is reduced to the channel frequency response $h_k[i, n]$, which follows a zero mean complex-valued Gaussian distributed random process with variance equal to 1.

C. MC-CDMA Receiver Model

At the receiver, the signal received by user k during the i -th symbol interval is first OFDM-demodulated by removing the guard interval and applying the fast Fourier transform (FFT) operation. After chip demapping, each resulting parallel stream is detected using a single user detection technique, which consists in a chip-per-chip equalization followed by a despreading [2][3]. Several equalization strategies have been considered in the literature: Orthogonality Restoring

Combining (ORC), Equal Gain Combining (EGC), Maximal Ratio Combining (MRC), and Minimum Mean Square Error Combining (MMSEC). After equalization and despreading, the parallel streams of decision variables are Parallel/Serial converted and then channel decoded to recover the transmitted binary information.

III. THEORETICAL ANALYSIS

A. Problem Formulation

The MAI term in the decision variable after equalization and despreading can be expressed similarly as in [3]:

$$MAI_k = \sum_{j \neq k} MAI_{kj} = \sum_{j \neq k} d_j \sum_{n=0}^{SF-1} c_j[n] \overline{c_k[n]} \rho_k[n] \quad (1)$$

where $\rho_k[n]$ is the real equalized channel coefficient for the n -th chip. Thanks to wide-sense stationary channel, we show that the power of the mutual interference MAI_{kj} can be exactly expressed as:

$$\sigma_{kj}^2 = \frac{1}{SF} \Gamma_{\rho_k}[0] + 2 \sum_{\ell=1}^{SF-1} \text{Re}(ACF_{kj}[\ell]) \Gamma_{\rho_k}[\ell] \quad (2)$$

where $\Gamma_{\rho_k}[\ell]$ is the statistical correlation function of ρ_k defined as $\Gamma_{\rho_k}[\ell] = E\{\rho_k[n] \rho_k[n-\ell]\}$, and ACF_{kj} stands for the aperiodic correlation function of the sequence resulting from the component-wise product $c_{kj}[n] = c_j[n] \overline{c_k[n]}$:

$$ACF_{kj}[\ell] = \sum_{n=0}^{SF-\ell-1} c_{kj}[n] \overline{c_{kj}[n+\ell]} \quad (3)$$

The total MAI power for user k can therefore be written as:

$$\sigma_k^2 = \sum_{j \neq k} \sigma_{kj}^2 \quad (4)$$

The second summation term in (2) expresses the influence of the couple (k,j) of interfering sequences on the mutual interference power due to the equalized channel correlation. In particular, when the equalized channel coefficients are uncorrelated, the mutual interference power becomes independent of the couple (k,j) since (2) reduces to:

$$\sigma_{kj}^2 \approx \frac{1}{SF} \left(\Gamma_{\rho_k} [0] - E\{\rho_k\}^2 \right) \quad (5)$$

In the same way, in the case of highly correlated channel coefficients, orthogonality among the spreading sequences is almost preserved since (2) converges to zero. Thus, whenever we approach these two extreme cases of uncorrelated and highly correlated channels, the influence of the couple (k,j) on the mutual MAI power becomes negligible. Consequently, the maximum variation of the mutual interference power with respect to the couple (k,j) of interfering sequences occurs between these two extreme cases of uncorrelated and highly correlated channels. This variation is as large as the dependency of the aperiodic correlation function ACF_{kj} on the sequence c_{kj} is high. For Walsh-Hadamard (WH) and Fourier sequences, ACF_{kj} varies drastically with respect to the sequence c_{kj} , whereas for Pseudo-Noise (PN) maximum length sequences, this variation is found to be much less significant. When the spreading sequences have a great influence on the MAI power, selecting the optimal subset of K sequences with minimum MAI powers can significantly improve the MC-CDMA system performance. The next section deals with the optimal spreading sequence selection issue.

B. Spreading Sequence Selection

Let $\Omega = \{1, \dots, N\}$ be the set of indexes of all possible spreading sequences and $X = \{x_1, \dots, x_K\}$ a subset of K elements in Ω . The user k is assigned the sequence $x_k \in \Omega$ instead of sequence k for the natural order. The optimal subset X_{opt} of the K sequences with minimum MAI powers is defined as:

$$X_{opt} = \left\{ x_i / \forall i = 1 \dots K, \forall X, \forall x_j \in X, \sigma_{x_i}^2 \leq \sigma_{x_j}^2 \right\} \quad (6)$$

The optimal subset as defined in (6) generally will not exist. In order to resolve this problem, one may define a scalar cost function $f(X)$ on the K -dimensions space, and then run the following exhaustive algorithm in order to find an optimal subset X^* such that:

$$X^* = \arg \min_{X \subseteq \Omega} f(X) \quad (7)$$

This algorithm has the main drawback of high computational costs. Indeed, it requires checking all the possible subsets of K elements in Ω in order to extract the optimal subset X^* that minimizes $f(X)$. An adequate choice of $f(X)$ is:

$$f(X) = \max_{x_k \in X} \left\{ \sigma_{x_k}^2 \right\} \quad (8)$$

In order to reduce the high computational costs of the exhaustive algorithm without significant loss in performance, the following recursive algorithm is proposed:

1) Step 1: Input parameters

The equalized channel correlations $\{\Gamma_{\rho_k}[\ell]\}$, the family of spreading sequences C and the system load K are provided.

2) Step 2: Repeat for recursion $n = 1 \dots N$

The index x_1 of the first sequence is set to n . The index x_2 of the second sequence is chosen such that:

$$x_2 = \arg \min_{x_j \in \Omega, x_j \neq x_1} \left\{ \sigma_{x_1 x_j}^2 \right\} \quad (9)$$

The index x_k of the k -th sequence is chosen such that:

$$x_k = \arg \min_{x_j \in \Omega, x_j \neq x_1 \dots x_{k-1}} \left\{ \max_{x_i = x_1 \dots x_{k-1}} \left\{ \sigma_{x_i x_j}^2 \right\} \right\} \quad (10)$$

Once the indexes of the K spreading sequences are obtained, they are stored in the sub-set X_n .

3) Step 3: Optimal sub-set X^*

The optimal sub-set is chosen such that:

$$X^* = \arg \min_{X_n, n=1 \dots N} \left\{ \max_{x_j \in X_n} \left\{ \sigma_{x_j}^2 \right\} \right\} \quad (11)$$

The complexity of this algorithm is significantly lower than that resulting from (7). Indeed, if we consider that finding the minimum or the maximum value of any set of elements corresponds to one operation, we can show that the number of operations required for the recursive algorithm is:

$$N_{op} = O(N^2 K) \quad (12)$$

This is much less than the number of operations required for the exhaustive algorithm, which is in the order of $\binom{N}{K}$.

The optimality of the recursive algorithm is quite difficult to derive analytically. However, simulations show that the performance of the recursive algorithm are very close to that of the exhaustive algorithm.

C. Practical Aspects

As we can see from the list of input parameters, the recursive algorithm requires the knowledge of the equalized channel correlations for the K active users. This information may not be easy to provide in practice. Thus, making some assumptions on these correlation functions will be helpful to practical implementation of the recursive algorithm. For instance, one may consider only one typical model of the equalized channel correlations for all K spreading sequences. This correlation can be derived from a typical power delay profile that models in average the multi-path channel in a specific environment. This can be justified when the K spreading sequences belong to the same active user, i.e. multi-code transmission, or when all sequences belong to active users in the same environment. Another way to deal with this

problem consists in making some simplified assumptions on the channel correlation model in order to extract a simplified metric requiring only basic channel knowledge. The aim is that the recursive algorithm can be run with the only knowledge of the family of spreading sequences, the system load and some basic parameters of the channel correlation. For instance, similarly to [7], if we assume only first order channel correlation, (2) reduces to:

$$\sigma_{kj}^2 \approx \frac{VAR(\rho_k)}{SF} + 2 \operatorname{Re}(ACF_{kj}[1]) (\Gamma_{\rho_k}[1] - E\{\rho_k\}^2) \quad (13)$$

Consequently, the mutual interference power becomes a monotonic increasing function of $ACF_{kj}[1]$. Thus, $ACF_{kj}[1]$ can be used instead of the mutual interference power to run the recursive algorithm. Note that $ACF_{kj}[1]$ can be directly related to the number of transitions in the sequence c_{kj} [7].

IV. PERFORMANCE EVALUATION

The propagation environment is modeled with the urban ETSI BRAN channel E with approximately 4MHz coherence bandwidth [8]. This channel model is used for all active users, and therefore all active users have the same equalized channel correlation. Without loss of generality, only spreading along the frequency dimension is considered with and without frequency interleaving. When frequency interleaving is active, the correlation between the channel coefficients becomes very low. This is referred to as the uncorrelated channel coefficients (UCC) scenario. Otherwise, the level of channel correlation is significant, and this is referred to as the correlated channel coefficients (CCC) scenario. The most relevant simulation parameters are summarized in Table 1.

Table 1. Simulation parameters.

Occupied bandwidth	54 MHz
Number of sub-carriers	480
FFT size	512
Spreading factor SF	31, 32
Spreading sequence	WH, Fourier, PN
Number of users K	From 1 to SF
Data modulation	QPSK
Detection technique	Single user detection
Equalization strategy	EGC
(E_b/N_0) ratio	10 dB
Channel model	ETSI BRAN E

Figure 2 depicts the mutual MAI powers from all interfering sequences sorted in the ascending order. As it can be observed from Figure 2, in the UCC scenario, the mutual MAI power exhibits slight variation with respect to the interfering spreading sequence. This is true independently of the family of spreading sequences. In the CCC scenario however, a large variation can be observed for WH and Fourier orthogonal sequences, whereas PN m -sequences still have no influence on the mutual MAI power. This means that the aperiodic correlation function ACF_{kj} is strongly dependent on the sequence $\{c_{kj}\}$ for WH and Fourier sequences, while it is not the case for PN m -sequences. Thus, as discussed in

Section III.A, when the interfering sequences have a great influence on the mutual MAI power, selecting the optimal subset of K less mutually interfering sequences can significantly improve the system performance.

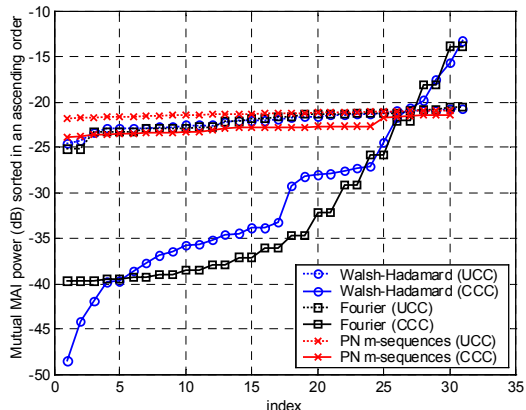


Figure 2: Mutual MAI powers sorted in the ascending order for lowly and significantly correlated channels.

Let us now move to the problem of spreading sequence selection. Five spreading sequence selections are considered for WH and Fourier sequences in the CCC scenario. The first selection is obtained from the recursive algorithm described in Section III.B where the real equalized channel correlation is considered at the input. The second selection is also obtained from the recursive algorithm where the mutual MAI power metric is replaced by the first order aperiodic correlation $ACF_{kj}[1]$ as described in (13). The third selection chooses the sequences in their natural order, whereas the fourth selection chooses them randomly among the SF available sequences. Finally, the last selection is obtained from the recursive algorithm by selecting the most interfering sequences instead of the less interfering ones (max instead of min in (9), (10) and (11)). This last one is a bad sequence selection procedure. The system performance is measured by the minimum achieved SINR over all active users since it is the one that predominantly affects the system average bit error rate (BER). Furthermore, the minimum achieved SINR is the most likely to produce a link outage and consequently it strongly affects the outage-based system capacity.

In Figure 3, Walsh-Hadamard sequences are considered, whereas Fourier sequences are considered in Figure 4. From Figure 3, it can be seen that the first, second and third selections give the same result. This means that one should choose Walsh-Hadamard sequences in the natural order to obtain the best system performance. This also means that selecting the sequences by using the first order aperiodic correlation criterion (cf. (13)) does not lead to any loss in performance. When comparing to the bad selection, we can observe an important gain ranging from approximately 1.5 to 3.25 dB up to 16 users. For higher system loads, much less significant gains (0.25 to 0.75 dB) are achieved. The highest achieved gain is found near 3.25 dB at system load $K = 8$.

This means that when comparing to different random selections, the gain ranges from 0 to 3.25 dB. For instance, for the random selection depicted in Figure 3, the maximum achieved gain is near 2.5 dB at system load $K = 8$.

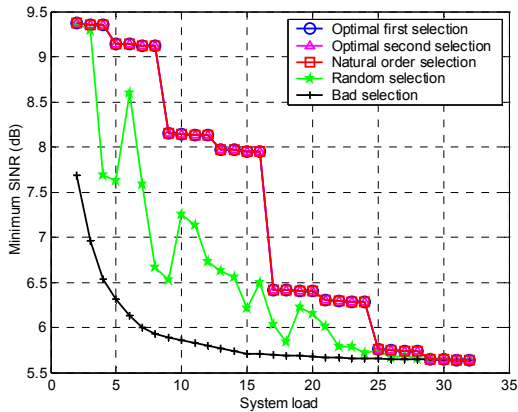


Figure 3: Minimum SINR versus System load for different selection procedures of Walsh-Hadamard sequences.

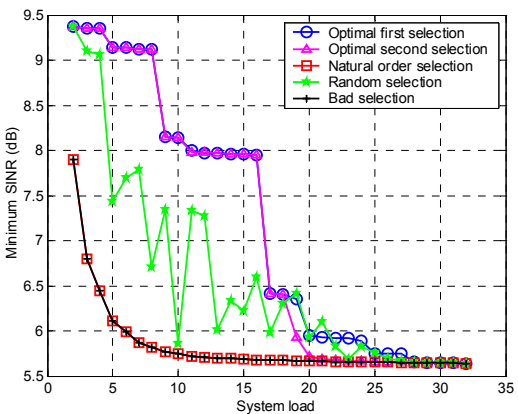


Figure 4: Minimum SINR versus System load for different selection procedures of Fourier sequences.

Figure 4 illustrates the five selections for Fourier spreading sequences. From Figure 4, it can be observed that the first and second selections provide the same result up to 18 users. For higher system loads, a non significant gain of approximately 0.25 dB is achieved by the first selection. This means that selecting the sequences by using the first order aperiodic correlation $ACF_{kj}[1]$ metric provides quite the same gains as when using the real mutual MAI power. In contrast to WH sequences, here, the natural order selection gives the same result as the bad selection. Thus, one should avoid selecting the Fourier sequences in the natural order. When comparing to bad selection, we can observe an important gain ranging from 1.5 to 3.5 dB up to 16 users. This means that the maximum gain that can be achieved over random selections is around 3.5 dB at system load $K = 8$. The gain achieved over the random selection depicted in Figure 4 is approximately 2.5 dB at system load $K = 8$.

Finally, we should point out that the values of the gain achieved by optimal sequence selections varies with respect to

the equalization strategy and the spreading factor value. Indeed, it has been observed that the gain is higher for MRC equalization and lower for MMSEC equalization than EGC equalization. Besides, the gain becomes more important when increasing the spreading factor.

V. CONCLUSION

In this paper, we have analyzed the impact of the spreading sequences on the MC-CDMA system performance. We have shown that between the extreme cases of uncorrelated and highly correlated channels, the spreading sequences have a great influence on the MAI power for Walsh-Hadamard and Fourier sequences. A recursive algorithm has been then proposed to adequately select the spreading sequences for a given system load. This algorithm has been shown to provide significant gains up to 3.5 dB in terms of minimum achieved SINR when comparing to bad selection for significantly correlated channels. The algorithm implementation requires knowledge of the equalized channel correlations of all active users, which may be difficult to provide in practice. However, it has been shown that assuming only first order channel correlation allows us to perform the algorithm off-line, i.e. with the only knowledge of the family of spreading sequences and the system load, and this without significant loss in performance. Finally, we should point out that the natural order selection is found to be the optimal selection for Walsh-Hadamard sequences, whereas it is the bad selection for Fourier sequences.

VI. REFERENCES

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