A pragmatic design of space-time-frequency coding

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***Abstract*—This paper presents a design methodology and resulting schemes for Space-Time-Frequency (STF) coding and linear precoding of downlink Multiple Input Multiple Output (MIMO) Orthogonal Frequency Division Multiplexing (OFDM) modulated wireless systems, based on practical hypotheses. In particular, the design takes into account the fact that the receiver implements STF Minimum Mean Square Error (MMSE) detector instead of traditional optimal receivers, which are probably too complex to address high spectral efficiencies. The design also takes into account that long term channel statistics is easily available at the transmitter side, while the receiver reasonably has access to instantaneous channel knowledge. The resulting schemes combine STF coding or linear precoding with long-term eigenbeam selection with a specific power loading. Numerical evaluation on correlated flat fading channel shows that they outperform a state-of-the-art scheme combining spatial multiplexing and Space Time Orthogonal Block Coding (STOBC) in terms of Bit Error Rate (BER) and other intermediate design criteria. Their low decoding complexity and good performance make them relevant for practical implementation.**

Keywords: MIMO, Space-Time-Frequency coding, OFDM, Downlink, Spatial correlation, Low complexity.

I. INTRODUCTION

Achieving large spectral efficiencies is a crucial requirement for future wireless systems that are expected to provide high throughput everywhere, whereas the allocated bandwidth will not necessarily increase a lot. Multiple Input Multiple Output (MIMO) antenna techniques are very good candidates to achieve such spectral efficiencies. Indeed, in theory the MIMO channel capacity has been shown to increase linearly with the minimum of the number of transmit antennas and receive antennas of the system under favorable spatially uncorrelated channel conditions [1].

Many schemes have since been designed to exploit this capacity, using the transmit and receive spatial diversity. For instance, Space Time Orthogonal Block Codes (STOBC) [2][3] aim at increasing the transmission reliability while keeping the information rate constant, by fully exploiting the spatial transmit diversity. On the contrary, spatial multiplexing schemes [4] aim at increasing the information rate at a given transmit power. They have been shown to approach the expected theoretical channel capacity provided that they are combined with a powerful channel code and that the receiver uses a nearly optimal iterative decoder based on nearly optimal spatial detector and nearly optimal channel decoder [5]. The advantage of STOBC is that the associated optimal receiver is very simple but their limitation is that they are not well suited to address large spectral efficiencies, as their coding rate is low. The advantage of spatial multiplexing schemes is their ability to provide large spectral efficiencies; however the receiver complexity is quite large even though suboptimal iterative receivers based on suboptimal and simple space-time detectors have been shown to offer a good performance complexity trade off [6]; besides, they require at least the same number of receive antennas as transmit antennas, which may not be practical in a downlink where the spatial dimension of the terminal is a constraint. Thus, intermediate space-time coding solutions have been designed to provide compromises between reliability increase and data rate increase, while accommodating different numbers of transmit and receive antennas, e.g. [7]. However, a recurrent issue is that these codes are designed assuming -explicitly or not- the use of optimal Maximum Likelihood (ML) or A Posteriori Probability (APP) decoding algorithms whose complexity is exponential with the spectral efficiency or at least polynomial in the power six in the case of suboptimal Sphere Decoding based algorithm [6][8]. Such algorithms are therefore very complex for practical implementation. Simple linear detectors such as Minimum Mean Square Error (MMSE) detector -or a polynomial approximation of it- are probably more attractive to address large spectral efficiencies. Although it is likely that codes designed with optimal receivers perform also well when decoded with suboptimal receivers, there is no guarantee of it. Thus, it seems more appropriate to design the codes taking into account the detector characteristics. This is one objective of this paper.

Moreover, as wireless systems generally implement low rate feedback or signaling channels, long term information on the physical channel is easily available at the transmitter side. Depending on the duplexing mode, the channel properties can even be estimated at the transmitter side without any dedicated feedback channel. Using long-term channel statistics to design space-time codes is not new, an example among others is [9], but the designs also implicitly assume optimal receivers.

From these considerations, this paper proposes the design of Space-Time-Frequency (STF) block codes under the hypothesis that long-term channel knowledge is available at the transmitter side and taking into account that MMSE detector is implemented at the receiver side. This is done in the context of OFDM modulation so that each subcarrier of each OFDM symbol experiences flat fading. Besides, we focus on the downlink, which is characterized by high spatial correlation at the transmitter side and low correlation at the receiver side.

The rest of the paper is organized as follows: chapter II provides the notations, system description and major hypotheses, chapter III presents the proposed design for linear STF precoding and STF coding, chapter IV provides numerical evaluation of the designed codes and chapter V draws conclusions.

II. NOTATIONS, SYSTEM DESCRIPTION AND HYPOTHESES

In the rest of the paper, vectors and matrices are represented with bold letters and scalars with italic letters. ^{t} represents the transpose, * the conjugate and ^{H} the hermitian transpose. For any matrix $\mathbf{M}$, $\mathbf{M}^R$ =Re($\mathbf{M}$) and $\mathbf{M}^I$ =Im($\mathbf{M}$). Tr is the trace operator. N_t and N_r are respectively the number of transmit and receive antennas.

A. Transmitter

We consider the design of STF block codes spanning a time dimension of N_{time} OFDM symbols and a frequency dimension of N_{freq} subcarriers. Thanks to OFDM modulation, the time and frequency dimensions are completely equivalent in the sense that they ideally remain orthogonal during a transmission. Therefore we do not make any distinction between these in the sequel and denote $N=N_{time}*N_{freq}$ the time and/or frequency dimensions spanned by the code. When N_{freq}=1, the code is a Space-Time (ST) code and when N_{time}=1 the code is a Space-Frequency (SF) code. We refer to the general STF denomination in the rest of the paper.

The STF block encoder encodes input Q*1 vectors $\mathbf{S}$ made of Q normalized modulated complex symbols. True STF codes will separately encode the real part and the imaginary part of $\mathbf{S}$, while linear STF precoders will perform a matrix multiplication of the complex vector. The STF codeword can be expressed as $\mathbf{E_c S}^R+\mathbf{F_c S}^I$, where the complex N_tN*Q matrices $\mathbf{E_c}$ and $\mathbf{F_c}$ fully describe the STF code. This writing is equivalent to $\mathbf{C_c S}+\mathbf{D_c S}^*$ where $\mathbf{C_c}$ and $\mathbf{D_c}$ are complex matrices encoding respectively $\mathbf{S}$ and $\mathbf{S}^*$. In the case of linear STF precoding, the matrix $\mathbf{D_c}$ is set to $\mathbf{0}$ and the linear STF precoder codeword becomes $\mathbf{C_c S}$.

B. MIMO OFDM channel

Thanks to OFDM modulation, for each pair of transmit and receive antenna, each subcarrier experiences flat fading where the related channel coefficient is equal to the channel frequency response at the subcarrier frequency, given by the corresponding sample of the FFT of the channel impulse response between the pair of antennas.

The MIMO channel impulse response is given by:

$$\mathbf{H}(t,\tau)=\sum_{p=1}^{Np}\mathbf{H}_p(t)\delta(\tau-\tau_p) \tag{1}$$

where t represents the time, τ the delay, Np is the number of multipaths components (also called taps), $\mathbf{H}_p(t)$ is a N_r*N_t complex random matrix representing the MIMO response of the p^{th} tap at time t. In general $\mathbf{H}_p(t)$ can be modeled as:

$$\mathbf{H}_p(t)=\alpha_p\mathbf{A}_p^H\mathbf{G}_p(t)\mathbf{B}_p^* \tag{2}$$

where α_p is the normalized tap amplitude, $\mathbf{G}_p(t)$ is a normalized i.i.d. complex centered gaussian distributed matrix, i.e. the distribution of each element of $\mathbf{G}_p(t)$ is $\left(1/\sqrt{2}\right)\left(N(0,1)+jN(0,1)\right)$, $\mathbf{A}_p$ and $\mathbf{B}_p$ are square roots of the spatial correlation matrices for tap p respectively at the receiver side and at the transmitter side, denoted $\mathbf{R}_{Rx,p}$ and $\mathbf{R}_{Tx,p}$: $\mathbf{A}_p^H\mathbf{A}_p=\mathbf{R}_{Rx,p}$, $\mathbf{B}_p^H\mathbf{B}_p=\mathbf{R}_{Tx,p}$. The writing (2) confers a so-called kronecker structure to the channel matrix $\mathbf{H}_p$ [10], which is generally valid, except for the Line Of Sight (LOS) component and for the so-called keyhole effect [11].

In this paper, we choose to focus on the downlink and make the assumption that the receiver is surrounded by many scatterers covering a large angular spread, so that the receive spatial correlation is close to identity for each tap. In this case we show that the correlation matrix of the frequency response of the channel is:

$$E_{\mathbf{H}}\left(\mathbf{H}'(t,f)\mathbf{H}'^H(t,f)\right)=\underbrace{\left(\sum_{p=1}^{Np}\left|\alpha_p\right|^2\mathbf{R}_{Tx,p}\right)}_{\mathbf{R}_{Tx}}\otimes I \tag{3}$$

where $\mathbf{H'}(t,f)$ is the vector made of the column vectors of the N_r*N_t frequency response matrix $\mathbf{H}(t,f)$ stacked on top of each other. This shows that the flat fading channel of each subcarrier also has a kronecker structure. We call $\mathbf{R}_{Tx}$ the resulting transmit correlation matrix and $\mathbf{B}$ its square root:

$$\mathbf{B}^H\mathbf{B}=\mathbf{R}_{Tx} \tag{4}$$

Thus, the channel matrix is modelled as:

$$\mathbf{H}(t,f)=\mathbf{G}(t,f)\mathbf{B}^* \tag{5}$$

where $\mathbf{G}(t,f)$ is a normalized i.i.d. complex centered gaussian distributed matrix. In the rest of the document, the dependency of $\mathbf{H}$ and $\mathbf{G}$ on t and f is removed to simplify the notations and because the channel is assumed to be stationary (in time and frequency) on the range of a STF codeword. Thus ,the channel seen by a STF codeword of length N is:

$$\mathbf{H}'=\begin{pmatrix}\mathbf{GB}^* & \cdots & \mathbf{0}\\ \vdots & \ddots & \vdots\\ \mathbf{0} & \cdots & \mathbf{GB}^*\end{pmatrix}=\mathbf{I}_N\otimes\mathbf{H}=\mathbf{I}_N\otimes\mathbf{GB}^* \tag{6}$$

C. MMSE receiver

We assume that the receiver has instantaneous channel knowledge, which is commonly implemented by appropriate pilot symbols.
In the case of linear STF precoding, the received signal can be expressed as:

$$\mathbf{R}' = \mathbf{H}'\mathbf{C}_c\mathbf{S} + \mathbf{v}' \qquad (7)$$

, where **R'** is the N_rN*1 received complex vector, made of N vectors of size N_r*1 stacked on top of each other, each corresponding to a time and/or frequency index i=1,..,N. Similarly **ν'** is a N_rN*1 vector of i.i.d. AWGN complex samples of variance σ^2.
The Minimum Mean Square Error (MMSE) estimate $\hat{\mathbf{S}}$ of **S** is given by:

$$\hat{\mathbf{S}} = \mathbf{C}_\mathbf{c}^{\mathrm{H}}\mathbf{H}'^{\mathrm{H}}\left(\mathbf{H}'\mathbf{C}_\mathbf{c}\mathbf{C}_\mathbf{c}^{\mathrm{H}}\mathbf{H}'^{\mathrm{H}} + \sigma^2\mathbf{I}_{NrN}\right)^{-1}\mathbf{R}' \qquad (8)$$

In the case of STF coding, we prefer the following writing involving only real matrices:

$$\underbrace{\begin{bmatrix}\mathbf{R}'^{\mathrm{R}} \\ \mathbf{R}'^{\mathrm{I}}\end{bmatrix}}_{\tilde{\mathbf{R}}} = \underbrace{\begin{bmatrix}\mathbf{H}'^{\mathrm{R}} & -\mathbf{H}'^{\mathrm{I}} \\ \mathbf{H}'^{\mathrm{I}} & \mathbf{H}'^{\mathrm{R}}\end{bmatrix}}_{\tilde{\mathbf{H}}}\underbrace{\begin{bmatrix}\mathbf{E}_\mathbf{c}^{\mathrm{R}} & \mathbf{F}_\mathbf{c}^{\mathrm{R}} \\ \mathbf{E}_\mathbf{c}^{\mathrm{I}} & \mathbf{F}_\mathbf{c}^{\mathrm{I}}\end{bmatrix}}_{\tilde{\mathbf{C}}}\underbrace{\begin{bmatrix}\mathbf{S}^{\mathrm{R}} \\ \mathbf{S}^{\mathrm{I}}\end{bmatrix}}_{\tilde{\mathbf{S}}} + \underbrace{\begin{bmatrix}\mathbf{v}'^{\mathrm{R}} \\ \mathbf{v}'^{\mathrm{I}}\end{bmatrix}}_{\tilde{\mathbf{v}}} \qquad (9)$$

The MMSE estimate of $\tilde{\mathbf{S}}$ is:

$$\hat{\tilde{\mathbf{S}}} = \tilde{\mathbf{C}}^t\tilde{\mathbf{H}}^t\left(\tilde{\mathbf{H}}\tilde{\mathbf{C}}\tilde{\mathbf{C}}^t\tilde{\mathbf{H}}^t + \sigma^2\mathbf{I}_{2NrN}\right)^{-1}\tilde{\mathbf{R}} \qquad (10)$$

III. Code Design

The design below is detailed in the case of linear STF precoding for the sake of simplicity, but the results are also provided for STF coding.

A. *Linear STF precoder design*

Starting from (8), the residual MSE after detection is found as:

$$\mathrm{MSE} = \sigma^2\mathrm{Tr}\left(\left(\mathbf{C}_\mathbf{c}^{\mathrm{H}}\mathbf{H}'^{\mathrm{H}}\,\mathbf{H}'\mathbf{C}_\mathbf{c} + \sigma^2\mathbf{I}_Q\right)^{-1}\right) \qquad (11)$$

We now seek the code such that:

$$\mathbf{C}_\mathbf{c} = \underset{\mathbf{C}_\mathbf{c}}{\mathrm{argmin}}\left(E_\mathbf{H}(\mathrm{MSE})\right) \text{ subject to } \mathrm{Tr}\left(\mathbf{C}_\mathbf{c}\mathbf{C}_\mathbf{c}^{\mathrm{H}}\right) = P \qquad (12)$$

, where P is the average total transmit power constraint. The problem lies in expressing $E_\mathbf{H}(\mathrm{MSE})$, which is the expectation of the MSE with respect to the fast fading process. To do this, we first assume that σ^2 is small (the SNR is large), which is a reasonable hypothesis, and hence approximate the MSE with the first order term in σ^2:

$$\mathrm{MSE} \approx \sigma^2\mathrm{Tr}\left(\left(\mathbf{C}_\mathbf{c}^{\mathrm{H}}\mathbf{H}'^{\mathrm{H}}\,\mathbf{H}'\mathbf{C}_\mathbf{c}\right)^{-1}\right) \qquad (13)$$

When the time and/or frequency dimension is 1 (N=1), **H'** reduces to **H** according to (6), and we observe that the matrix $\left(\mathbf{C}_\mathbf{c}^{\mathrm{H}}\mathbf{H}'^{\mathrm{H}}\,\mathbf{H}'\mathbf{C}_\mathbf{c}\right)^{-1} = \left(\mathbf{C}_\mathbf{c}^{\mathrm{H}}\mathbf{B}^t\mathbf{G}^{\mathrm{H}}\mathbf{G}\mathbf{B}^*\mathbf{C}_\mathbf{c}\right)^{-1}$ has an inverse central complex Wishart distribution with N_r degrees of freedom and associated covariance matrix $\mathbf{\Sigma} = \mathbf{C}_\mathbf{c}^{\mathrm{H}}\mathbf{B}^t\mathbf{B}^*\mathbf{C}_\mathbf{c} = \mathbf{C}_\mathbf{c}^{\mathrm{H}}\mathbf{R}_{Tx}^*\mathbf{C}_\mathbf{c}$, denoted ICW$_{Nt}$(**Σ**,$N_r$). In this case, it has been shown that [12]:

$$E_\mathbf{H}\left(\mathbf{C}_\mathbf{c}^{\mathrm{H}}\mathbf{H}'^{\mathrm{H}}\,\mathbf{H}'\mathbf{C}_\mathbf{c}\right)^{-1} = \frac{1}{N_r - N_t}\mathbf{\Sigma}^{-1}, \quad N_r > N_t \qquad (14)$$

, which is of course still valid when we further apply the trace operator. This result has been shown to be valid in the case of N=1, but the following approximation still holds when $N > 1$ and $N_r \le N_t$, provided that the code rate $R = Q/N_rN$ is smaller than 1 (this ratio always needs to be smaller than 1 for the receiver to be feasible):

$$E_\mathbf{H}\left(\left(\mathbf{C}_\mathbf{c}^{\mathrm{H}}\mathbf{H}'^{\mathrm{H}}\,\mathbf{H}'\mathbf{C}_\mathbf{c}\right)^{-1}\right) \approx \alpha\,\mathbf{\Sigma}^{-1} = \alpha\left(\mathbf{C}_\mathbf{c}^{\mathrm{H}}\left(\mathbf{I}_N \otimes \mathbf{R}_{Tx}^*\right)\mathbf{C}_\mathbf{c}\right)^{-1} \qquad (15)$$

, where α is constant proportionality factor that does not depend on the choice of $\mathbf{C_c}$ but only on the dimensions of the problem. Therefore, using the approximations (13) and (15), the problem (12) can be formulated as finding $\mathbf{C_c}$ such that

$$\mathbf{C}_\mathbf{c} = \underset{\mathbf{C}_\mathbf{c}}{\mathrm{argmin}}\left(\mathrm{Tr}\left(\left(\mathbf{C}_\mathbf{c}^{\mathrm{H}}\left(\mathbf{I}_N \otimes \mathbf{R}_{Tx}^*\right)\mathbf{C}_\mathbf{c}\right)^{-1}\right)\right) \qquad (16)$$
$$\text{subject to } \mathrm{Tr}\left(\mathbf{C}_\mathbf{c}\mathbf{C}_\mathbf{c}^{\mathrm{H}}\right) = P$$

We use the following Singular Value Decomposition (SVD) of $\mathbf{C_c}$:

$$\mathbf{C}_\mathbf{c} = \mathbf{P}\begin{pmatrix}\mathbf{\Gamma}^{1/2} \\ \mathbf{0}\end{pmatrix}\mathbf{V}^{\mathrm{H}} \qquad (17)$$

where **P** is a unitary N_tN* N_tN matrix, **V** is a unitary Q*Q matrix and **Γ** is a positive diagonal Q*Q matrix. We use the following Eigenvalue Decompostion (EVD) of $\mathbf{R}_{Tx}^*$:

$$\mathbf{R}_{Tx}^* = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^{\mathrm{H}} = \mathbf{B}^t\mathbf{B}^* \qquad (18)$$

so that

$$\mathbf{I}_N \otimes \mathbf{R}_{Tx}^* = \underbrace{\left(\mathbf{I}_N \otimes \mathbf{U}\right)}_{\mathbf{U}'}\underbrace{\left(\mathbf{I}_N \otimes \mathbf{\Lambda}\right)}_{\mathbf{\Lambda}'}\underbrace{\left(\mathbf{I}_N \otimes \mathbf{U}^{\mathrm{H}}\right)}_{\mathbf{U}'^{\mathrm{H}}} \qquad (19)$$

, where the compact writing with **U'** and **Λ'** authorizes that the eigenvalues in $\mathbf{I}_N \otimes \mathbf{\Lambda}$ are reordered arbitrarily and the eigenvectors (column vectors) in $\mathbf{I}_N \otimes \mathbf{U}$ are reordered accordingly.
The solution $\mathbf{C_c}$ to (16) is found as:

$$\mathbf{C}_\mathbf{c} = \sqrt{\frac{P}{\mathrm{Tr}\left(\mathbf{\Lambda}'^{-1/2}_Q\right)}}\,\mathbf{U}'\begin{pmatrix}\mathbf{\Lambda}'^{-1/4}_Q \\ \mathbf{0}\end{pmatrix}\mathbf{V}^{\mathrm{H}} \qquad (20)$$

$\mathbf{\Lambda}'_Q$ is a diagonal matrix made of the Q largest values among the N_tN diagonal values of **Λ'** and the first Q column vectors in **U'** are those associated with $\mathbf{\Lambda}'_Q$ in (19) (the remaining N_tN – Q vectors of **U'** have no importance as they are not used). Equivalently, $\mathbf{C_c}$ can be written as:

$$\mathbf{C}_\mathbf{c} = \sqrt{P/\mathrm{Tr}\left(\mathbf{\Lambda}'^{-1/2}_Q\right)}\,\mathbf{U}''\mathbf{\Lambda}'^{-1/4}_Q\,\mathbf{V}^{\mathrm{H}} \qquad (21)$$

, where **U''** is a N_tN * Q matrix made of the first Q columns of **U'**.
In (20), **U'** and $\mathbf{\Lambda}'_Q$ are fully defined, **V** is only required to be unitary. However, whereas any unitary matrix **V** gives the

same average value of the MSE with respect to the fast fading process, the choice of **V** influences the resulting Bit Error Rate (BER). Indeed, the choice of **V** influences the residual SINR per dimension of the estimate $\hat{\mathbf{S}}$ after MMSE equalization, and hence the error probability per dimension. **V** can therefore be chosen so as to maximize the minimum average SINR per dimension, which can be formulated as the following minimax problem:

$$\begin{cases}\mathbf{V} = \arg\max_{\mathbf{V}}\left(\min_{i\in\{1,\dots,Q\}} \mathrm{E}_{\mathbf{H}}\left(\mathrm{SINR}_i\right)\right) \\ \text{subject to } \mathbf{V}\mathbf{V}^{\mathrm{H}} = \mathbf{I}_Q\end{cases} \quad (22)$$

, where SINR_i is the SINR of the i^{th} component of $\hat{\mathbf{S}}$, which we express in the following. The estimated signal $\hat{\mathbf{S}}$ after encoding with the code (20), transmission over the MIMO channel and MMSE equalization can be written as:

$$\begin{aligned}\hat{\mathbf{S}} = &\left(\mathbf{I}_Q - \sigma^2\mathbf{V}\left(\mathbf{f}\left(\mathbf{G}^{\mathrm{H}}\mathbf{G}\right)+\sigma^2\mathbf{I}_Q\right)^{-1}\mathbf{V}^{\mathrm{H}}\right)\mathbf{S} \\ &+\mathbf{V}\left(\mathbf{f}\left(\mathbf{G}^{\mathrm{H}}\mathbf{G}\right)+\sigma^2\mathbf{I}_Q\right)^{-1}\mathbf{g}\left(\mathbf{G}\right)\mathbf{v}'\end{aligned} \quad (23)$$

, where $\mathbf{f}\left(\mathbf{G}^{\mathrm{H}}\mathbf{G}\right)$ and $\mathbf{g}(\mathbf{G})$ are matrix functions of respectively $\mathbf{G}^{\mathrm{H}}\mathbf{G}$ and $\mathbf{G}$, which is the centered normal i.i.d. matrix introduced in (5). We have the following relationship: $\mathbf{f}\left(\mathbf{G}^{\mathrm{H}}\mathbf{G}\right)=\mathbf{g}(\mathbf{G})\mathbf{g}^{\mathrm{H}}(\mathbf{G})$. The SINR of the i^{th} component is then given by:

$$\mathrm{SINR}_i = \frac{1}{\sigma^2\mathbf{V}_i\left(\mathbf{f}\left(\mathbf{G}^{\mathrm{H}}\mathbf{G}\right)+\sigma^2\mathbf{I}_Q\right)^{-1}\mathbf{V}_i^{\mathrm{H}}} - 1 \quad (24)$$

, where $\mathbf{V}_i$ is the i^{th} line of **V**. The problem (22) becomes:

$$\begin{cases}\mathbf{V} = \arg\max_{\mathbf{V}}\left(\min_{i\in\{1,\dots,Q\}} \mathrm{E}_{\mathbf{G}}\left(\frac{1}{\mathbf{V}_i\left(\mathbf{f}\left(\mathbf{G}^{\mathrm{H}}\mathbf{G}\right)+\sigma^2\mathbf{I}_Q\right)^{-1}\mathbf{V}_i^{\mathrm{H}}}\right)\right) \\ \text{subject to } \mathbf{V}\mathbf{V}^{\mathrm{H}} = \mathbf{I}_Q\end{cases} \quad (25)$$

Neglecting second order terms (in $(\sigma^2)^2$) that appear in the Taylor series expansion of SINR_i since σ^2 is small, it can be shown that the resulting term $\mathbf{V}_i\left(\mathbf{f}\left(\mathbf{G}^{\mathrm{H}}\mathbf{G}\right)\right)^{-1}\mathbf{V}_i^{\mathrm{H}}$ at the denominator has a central chi-square distribution. Besides, it can be shown that the expectation of the inverse of a chi-square distributed variable is proportional to the inverse of the expectation of the chi-square distributed variable and therefore that (25) can be reformulated as:

$$\begin{cases}\mathbf{V} \approx \arg\min_{\mathbf{V}}\left(\max_{i\in\{1,\dots,Q\}} \mathrm{E}_{\mathbf{G}}\left(\mathbf{V}_i\left(\mathbf{f}\left(\mathbf{G}^{\mathrm{H}}\mathbf{G}\right)\right)^{-1}\mathbf{V}_i^{\mathrm{H}}\right)\right) \\ \quad\approx \arg\min_{\mathbf{V}}\left(\max_{i\in\{1,\dots,Q\}}\left(\mathbf{V}_i\mathbf{\Lambda'}_Q^{-1/2}\mathbf{V}_i^{\mathrm{H}}\right)\right) \\ \text{subject to } \mathbf{V}\mathbf{V}^{\mathrm{H}} = \mathbf{I}_Q\end{cases} \quad (26)$$

This problem is a non-linear optimization problem:

- it can be solved numerically recursively and converges to different matrices **V** for different initial matrices but at the convergence all cost functions are equal,
- the cost function $\left(\mathbf{V}_i\mathbf{\Lambda'}_Q^{-1/2}\mathbf{V}_i^{\mathrm{H}}\right), i\in\{1,\dots,Q\}$ has all its values equal at the convergence and they are equal to those obtained with Hadamard and Discrete Fourier Transform (DFT) matrices, which are therefore local optima.

Thus, **V** can be chosen as a DFT matrix or a Hadamard matrix when Q is a power of 2.

B. STF code design

Similar reasoning, which we will not detail here, leads to the solution for STF coding. We first define $\tilde{\mathbf{B}}$ as:

$$\tilde{\mathbf{B}} = \begin{pmatrix}\mathbf{I}_N\otimes\mathbf{B}^{\mathrm{R}} & \mathbf{I}_N\otimes\mathbf{B}^{\mathrm{I}} \\ -\mathbf{I}_N\otimes\mathbf{B}^{\mathrm{I}} & \mathbf{I}_N\otimes\mathbf{B}^{\mathrm{R}}\end{pmatrix} \quad (27)$$

where **B** is defined in (4).

The STF code is found as:

$$\tilde{\mathbf{C}} = \sqrt{\frac{P}{\mathrm{Tr}(\tilde{\mathbf{\Lambda}}_{2Q}^{-1/2})}}\,\tilde{\mathbf{U}}\tilde{\mathbf{\Lambda}}_{2Q}^{-1/4}\mathbf{V}^{\mathrm{t}} \quad (28)$$

where P is the average total transmit power, $\tilde{\mathbf{\Lambda}}_{2Q}$ is a diagonal matrix made of the $2Q$ largest eigenvalues of $\tilde{\mathbf{B}}^{\mathrm{t}}\tilde{\mathbf{B}}$ and $\tilde{\mathbf{U}}$ is made of the related $2Q$ eigenvectors. **V** is an orthogonal matrix whose definition can be refined as in the previous chapter. In particular, Hadamard matrices when Q is a power of 2, or matrices obtained from DFT matrices are appropriate solutions.

IV. NUMERICAL RESULTS

The codes designed above are evaluated using Monte Carlo simulation on a narrowband flat fading channel taking into account the model of chapter II.B. The parameters are set as follows: N_t=4, N_r=2, N=2, Q=4. The input complex symbols are QPSK modulated.

The transmit correlation is computed from geometric path distribution. In the selected simulation scenario, the elementary paths leaving the transmitter have a laplacian distribution with an average angle of departure of 45° with respect to the bore sight and a standard deviation of 5°, which roughly gives the position and dimension of the main scatterer seen by the transmitter. The spacing between the antenna elements is half wavelength. These parameters are typical of an outdoor environment and lead to a rather high spatial correlation at the transmitter. The receiver has no spatial correlation.

The abscissa of the simulation curves is the average Eb/No, where Eb is the total transmitted energy per information bit and No is the noise power spectral density (No = $2\sigma^2$) at each receive antenna.

In Fig. 1 to 3, 3 codes are evaluated: the linear STF precoding and STF coding as described above, and a code made of the combination of spatial multiplexing with Alamouti's scheme [2]. This code consists in transmitting at time instant 1: S_1, S_2, S_3, S_4 on antennas 1 to 4, and at time instant 2: $-S^*_2$, S^*_1, $-S^*_4$,

S^*_3 on antennas 1 to 4. This code is known as providing full transmit diversity with a good coding gain. As it is optimally decoded with a MMSE receiver, it is not here as an example of code designed assuming an optimal receiver and decoded with a suboptimal decoder, but as an example of a very good code.

Fig. 1 shows the residual MSE vs. the Eb/No. The designed codes indeed provide a minimum residual MSE. This strengthens the approximation (15).

Fig. 2 shows the average minimum SINR over all Q dimensions after MMSE detection. The designed codes indeed provide the maximum minimum SINR, which is consistent with the design criterion (22) and somehow validates the approximation made in (26).

Fig. 3 shows the average BER. It is limited between 10^{-2} and 10^{-3} because this is sufficient to draw conclusions, all the more as channel coding is generally implemented to reduce the residual BER. The gain brought by the designed codes is significant (almost 4dB at a BER of 10^{-2}). According to the curves slopes, full transmit diversity is achieved. According to the curves relative positions, a good antenna (coding) gain is achieved. It is interesting to notice that, with the proposed design, STF coding does not bring any gain with respect to linear STF precoding.

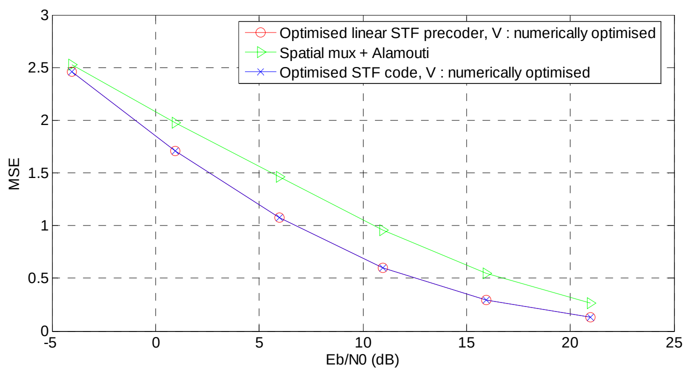

Figure 1: Average residual MSE after MMSE detection.

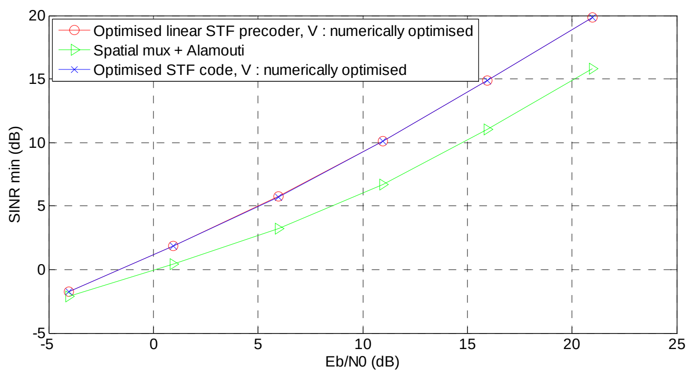

Figure 2: Average of min{$SINR_i$,i=1,..,Q} after MMSE detection.

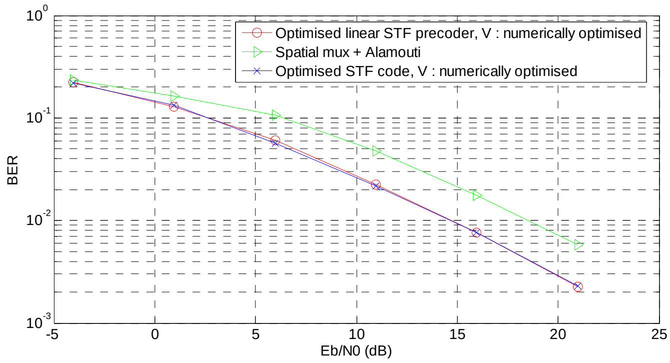

Figure 3: Average BER after MMSE detection.

V. CONCLUSIONS

This paper presents STF coding and linear precoding schemes for downlink MIMO OFDM systems, designed taking into account the complexity constraint of the receiver and making rather weak channel knowledge hypotheses. In particular, the receiver is assumed to implement a MMSE STF detector having lower complexity than traditionally assumed optimal detectors; besides, long-term channel knowledge is available at the transmitter while instantaneous channel knowledge is available at the receiver. The resulting schemes have been shown to perform well on spatially correlated channel. They are probably good candidates for practical implementation thanks to their low complexity and good performance.

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