Space-time coding techniques with bit-interleaved coded modulations for MIMO block-fading channels

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Abstract

The space-time bit-interleaved coded modulation (ST-BICM) is an efficient technique to obtain high diversity and coding gain on a block-fading MIMO channel. Its maximum-likelihood (ML) performance is computed under ideal interleaving conditions, which enables a global optimization taking into account channel coding. Thanks to a diversity upperbound derived from the Singleton bound, an appropriate choice of the time dimension of the space-time coding is possible, which maximizes diversity while minimizing complexity. Based on the analysis, an optimized interleaver and a set of linear precoders, called dispersive nucleo algebraic (DNA) precoders are proposed. The proposed precoders have good performance with respect to the state of the art and exist for any number of transmit antennas and any time dimension. With turbo codes, they exhibit a frame error rate which does not increase with frame length.

Index terms

Multiple antenna channels, bit-interleaved coded modulation, space-time coding, Singleton bound, interleaving

1 Introduction

The wide panel of today's wireless transmission contexts makes implausible the existence of a miraculous universal solution which always exhibits good performance with low complexity. Different scenarios (indoor, outdoor with low velocity, outdoor with high velocity) correspond to different amounts of time and frequency diversity. The success of the multi-carrier modulation as a solution for future wireless systems is in part due to the low receiver complexity even over large frequency bands. In this paper, we focus on an indoor environment and design a system approaching the optimal performance taught by information theory. In a wireless indoor environment, both time and frequency diversities may be poor due to small terminal velocity and possibly very short channel impulse response. These particularly tricky low-diversity channels are modelled as block-fading channels. Over low-diversity multiple-input multiple-output (MIMO) channels, space-time coding techniques often enable transmission with improved data rate and diversity, within a limit given by the rank of the MIMO system [1][18][19][21]. As a first step in providing increased data rates in future generations of indoor wireless local access networks (WLANs), we study how to appropriately choose the channel coding, the channel interleaving and the space-time coding.

For frame sizes of practical interest, coded modulations have to be considered since space-time codes employed with uncoded modulations exhibit a frame error rate (FER), which is dramatically degraded [23, Annex A]. Thus, we focus on the bit-interleaved coded modulation (BICM) structure, which is the concatenation of a channel encoder, an interleaver and a modulator. The analysis of the BICM maximum-likelihood (ML) performance is tractable and eases the coded modulation design. Furthermore, thanks to the interleaver, iterative processing at the receiver achieves quasi-ML performance with reduced complexity. On a MIMO channel, the BICM may be concatenated with a simple full-rate space division multiplexing scheme (SDM) [21]. In this paper, we improve performance of this space-time BICM (ST-BICM) by replacing the SDM by a more efficient full-rate linear space-time code: a linear precoding or equivalently a space-time spreading. Linear precoding is performed by multiplying the complex multiple-antenna signal by

a square complex space-time matrix. The space-time matrix enhances the diversity by mixing the symbols of different time periods and antennas together.

The choice of the ST-BICM structure may also be explained as follows: We aim at optimizing a full-rate space-time code based on linear precoding, taking into account the structure of the whole transmitter, which inevitably includes an error correction code, an interleaver and a symbol mapper. Usually, space-time codes are designed independently from the other elements of the transmitter. However, frames of bits are linked through the error correction code and optimizing the space-time code taking into account the whole transmitter is equivalent to optimizing a BICM concatenated with a space-time code, i.e., an ST-BICM.

On an ergodic channel, the achieved diversity order is equal to the code minimum distance multiplied by the number of receive antennas. In most cases, the minimum distance is high enough and increasing diversity through linear precoding does not bring much improvement. For a block-fading channel, the diversity is upperbounded by the number of channel realizations in a codeword multiplied by the number of transmit antennas and the number of receive antennas. Using the Singleton bound, we will exhibit an additional upperbound on the diversity order, which may be very limiting without precoding. Hence, in this paper, we will study ST-BICM with linear precoding, focusing on the block-fading channel and optimize the linear precoding using the ST-BICM ML performance in order to achieve full-diversity and maximum coding gain. First, we derive the coding gain of an ideal ST-BICM. It is related to the notion of Shannon code and sphere-hardening [38]. Indeed, the ideal Shannon code for additive white gaussian noise (AWGN) channels is located near a sphere, called the Shannon's sphere. Thanks to the interleaver, the squared Euclidean norm of BICM codewords has low variance, which implies that codewords lie close to the Shannon's sphere. The BICM may be seen as a quantization of this sphere, which should be as uniform as possible to maximise the size of Voronoi regions. On MIMO fading channels, the Shannon's sphere becomes a Shannon's ellipsoid [20] and BICM codewords are randomly

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located close to the ellipsoid. We show that the ideal BICM configuration maximizes the Voronoi region volume whatever the channel realization. We present a practical system that approaches the ideal BICM configuration including the so-called dispersive nucleo algebraic (DNA) precoder and compare its performance to the ideal ST-BICM performance. The DNA precoder exists for any numbers of space and time dimensions. We finally design a practical interleaver, which approximates the ideal interleaving conditions.

The paper is organized as follows: In section 2, the ST-BICM transmitter and the associated iterative receiver are presented. In section 3, we derive the analytical ML performance under ideal interleaving assumption for an ergodic channel without precoding and a block-fading channel with and without precoding. Using the Singleton bound, we show in section 4 that ideal interleaving conditions cannot be achieved on a block-fading channel with any kind of parameters and that linear precoding may be mandatory in some configurations. Section 5 describes the linear precoding optimization for a block-fading channel and section 6 the interleaver design for convolutional codes and its application to turbo codes. Finally, simulation results are presented in section 7, which confirm the behavior which was expected from the analytical study. Furthermore, they show the good performance of the DNA precoders and the advantage of using turbo codes to get a non-increasing FER when the frame size increases.

2 System model and notations

2.1 Transmitter scheme

The transmitter scheme is built from the following fundamental block concatenation: A binary error-correcting code $\mathcal{C}$ followed by a deterministic interleaver Π, a symbol mapper (e.g., for a quadrature amplitude modulation (QAM)), a full-rate space-time spreader S (i.e., a linear precoder) and a set of n_t transmit antennas. Fig. 1 illustrates the BICM transmitter structure.

Without loss of generality, we assume that the error correcting code $\mathcal{C}$ is a convolutional code with rate $R_{\mathcal{C}}$. The encoder associates with the input information word b the codeword $c \in \mathcal{C}$. Sequence b (resp. c) has length $K_{\mathcal{C}} L_{\mathcal{C}}$ (resp. $N_{\mathcal{C}} L_{\mathcal{C}}$) bits, where $L_{\mathcal{C}}$ is the codeword length in trellis branches. The interleaver Π, which scrambles the $L_{\mathcal{C}} N_{\mathcal{C}}$ coded bits, is a crucial function in the BICM structure, as it allows the receiver to perform iterative joint detection and decoding. Indeed, it ensures independence between extrinsic and a priori probabilities, in both the detector and the decoder. Furthermore, when maximum-likelihood (ML) decoding is tractable, interleaving prevents erroneous bits of a same error event from interfering to each other in the same precoded symbol. The interleaver Π may be pseudo-random (PR) or semi-deterministic with some deterministic constraints as described in section 6. In the symbol mapper, m consecutive interleaved coded bits are mapped together onto a modulation symbol, according to a bijection between bit vectors and modulation symbols called mapping or labeling. The number of modulation symbols is equal to $M = 2^m$. For each channel use, i.e., in each time period, the mapper reads $m n_t$ coded bits and generates n_t modulation symbols. To make the reading easier, the obtained n_t-dimensional constellation Ω will denote both the set of symbols and the set of binary labelings. All along this paper, we will consider QAM modulations as they achieve a good compromise between spectral efficiency (in bits/s/Hz or bits/dim) and performance. Moreover, with QAM modulation, the system is easily modeled using a lattice constellation structure [10], which gives access to the lattice theory toolbox, both for transmitter and receiver optimizations. We assume that the QAM modulation has unit energy. The linear precoder S spreads the QAM symbols over s time periods. It converts the $n_t \times n_r$ vector channel into an $N_t \times N_r$ vector channel, where $N_t = n_t s$ and $N_r = n_r s$. The $N_t \times N_t$ matrix S multiplies a vector of N_t QAM symbols $z_k = (z_{k,1}, z_{k,2}, \ldots, z_{k,N_t})$ at the mapper output, generating N_t symbols to be transmitted during s time periods. Vector z_k is the kth vector to be precoded. The precoder S spreads the transmitted symbols over a higher number of channel states to exploit diversity. S is normalized as follows:

$$\sum_{i=1}^{N_t} \sum_{j=1}^{N_t} S_{i,j}^2 = N_t \tag{1}$$

In this paper, we assume a block-fading channel with n_c distinct channel realizations during a codeword. We denote n_s the number of distinct channel realizations during a precoded symbol. To simplify notations, we assume that n_s divides n_c. We will call *channel state* the $1 \times n_r$ SIMO channel associated with one of the n_t transmit antennas and one of the n_c channel realizations. The channel experienced by precoded symbol k is represented by a $N_t \times N_r$ block-diagonal matrix H_k with s blocks of size $n_t \times n_r$. We assume that each of the n_s channel realizations is repeated s/n_s times. Matrix H_k is organized as follows:

$$H_k = \mathrm{diag}\left(H_k^{[1]}, \ldots, H_k^{[1]}, H_k^{[2]}, \ldots, H_k^{[2]}, \ldots, H_k^{[n_s]} \ldots, H_k^{[n_s]}\right) \tag{2}$$

where $H_k^{[t]}$ denotes the $n_t \times n_r$ complex matrix representing the t-th channel realization experienced by precoded symbol k. $H_k^{[t]}$ is repeated s/n_s times. Elements of $H_k^{[t]}$ are independent complex Gaussian variables with zero mean and unit variance. Let $\mathcal{H}$ denote the set of channel realizations observed during the transmission of a codeword. Thanks to the extended channel matrix, we write the channel input-output relation as:

$$y_k = x_k + \eta_k = z_k S H_k + \eta_k \tag{3}$$

where $y_k \in \mathbb{C}^{N_r}$ and each receive antenna is perturbed by an additive white complex Gaussian noise $\eta_{k,j}$, $j = 1 \ldots N_r$, with zero mean and variance $2N_0$. We define the signal-to-noise ratio E_b/N_0, where E_b is the total energy of an information bit at the receiver. Thanks to linear precoding, the $n_t \times n_r$ MIMO n_c-block-fading channel is converted into an $N_t \times N_r$ MIMO N_c-block-fading channel where $N_c = n_c/n_s$. If $n_s = 1$, the precoder experiences a quasi-static $n_t \times n_r$ MIMO channel. In the following, index k will be omitted if a single precoded symbol is considered and *precoding time period* will refer to a transmission over SH, i.e., over s time periods.

The concatenation of the binary error correcting code $\mathcal{C}$, the interleaver Π, the mapper Ω, the linear precoder S and the channel describes a global Euclidean code $\mathcal{C}_E$ which converts $L_{\mathcal{C}} K_{\mathcal{C}}$

information bits into a complex $L_\mathcal{C} N_\mathcal{C}/m$-dimensional point.

2.2   Iterative receiver scheme

An ideal BICM receiver would directly perform an ML decoding on the set $\mathcal{C}_E$ of transmitted codewords. However, it requires an exhaustive search among the $2^{K_c L_c}$ codewords, which is intractable. All existing receivers use the concatenated structure of the BICM to split the reception into several steps. In this paper, we assume perfect synchronization and channel estimation. Thus, the receiver, as depicted on Fig. 2, is divided in two main elements: a soft-input soft-output (SISO) APP QAM detector, which acts as a soft-output equalizer for both the space-time spreader and the MIMO channel, converting the received point y into information on the coded bits in the estimated coded sequence $\hat{c}$, and a SISO decoder for $\mathcal{C}$, improving the information on coded bits and estimating the information bit sequence $\hat{b}$. The depicted iterative joint detection and decoding process is based on the exchange of soft values between these two elements. The SISO detector computes extrinsic probabilities $\xi(c_\ell)$ on coded bits thanks to the conditional likelihoods $p(y_k/z)$ and the *a priori* probabilities $\pi(c_\ell)$ fed back from the SISO decoder:

$$\xi(c_\ell) = \frac{\sum_{z' \in \Omega(c_\ell=1)} \left[e^{-\frac{\|y_k - z' S H_k\|^2}{2N_0}} \prod_{r \neq \ell} \pi(c_r) \right]}{\sum_{z \in \Omega} \left[e^{-\frac{\|y_k - z S H_k\|^2}{2N_0}} \prod_{r \neq \ell} \pi(c_r) \right]} \tag{4}$$

where Ω is the Cartesian product $(M\text{-QAM})^{N_t}$, i.e., the set of all vectors z generated by the QAM mapper, $|\Omega| = 2^{m N_t}$. The subset $\Omega(c_\ell = 1)$, for $\ell = 0, 1, \ldots, m N_t - 1$, is restricted to the vectors z in which the ℓ-th coded bit is equal to 1. The detector independently computes the soft outputs for each precoding time period. At the first iteration, no a priori information is available at the detector input. Through the iterations, the a priori probability on constellation points computed from the probabilities fed back by the SISO decoder becomes more and more accurate. Ideal convergence is achieved when a priori probabilities provided by the decoder are perfect, i.e., equal to 0 or 1. The decoder uses a forward-backward algorithm [2], which computes the exact extrinsic probability using the trellis structure of the code.

3   Theoretical performance for ideally interleaved BICM

Heavy work has been made to estimate the frame or bit error rate of the BICM with ML decoding, in particular using Gaussian approximations or numerical integrations [5], but a closed-form expression of the pairwise error probability had not been derived yet. This section first describes an accurate computation of bit and frame error rates of BICM ML performance over ergodic MIMO channel with ideal interleaving and without precoding. A more detailed description of the derivation may be found in [22] and [23]. Under ideal interleaving condition, we are able to derive a closed form expression of the probability density of the log likelihood ratio (LLR) at the output of the detector and then a closed form expression of the pairwise error probability at the output of the decoder. It is then straightforward to use well-known techniques to estimate the bit or frame error rate of a coded modulation from pairwise error probability. This subject has been extensively discussed for coded modulations over AWGN channels. Examples are the union bound on the transfer function of a convolutional code and the more accurate tangential sphere bound [35] for spherical constellations. Subsequently, we extend the study to the block-fading MIMO channel when linear precoding is used at the transmitter. Note that the method is also valid for correlated MIMO channels. We extract from the bit error rate expression some design criteria on BICM precoder, interleaver, and error correcting code.

3.1   Ideal interleaving condition

The evaluation of the bit error rate (BER) or frame error rate (FER) of a coded modulation is usually based on the derivation of an upper bound on the actual performance obtained by a balanced summation of pairwise error probabilities. Each pairwise error probability involves the Euclidean distance between two codewords with a Hamming distance w.

With an $n_t \times n_r$ MIMO block-fading channel with n_c blocks, the minimum diversity recovered at the detector output, and thus at the decoder output, is always equal to the reception diversity n_r. Let us consider an error event with w erroneous bits. Assume that the maximum diversity

order is Υ_{max}. If $w \geq \Upsilon_{max}$, we achieve full diversity if each of the Υ_{max} independent fading random variables is experienced by at least one bit among w. In a precoding time period k in which at least an erroneous bit is transmitted, the transmitted and competing points are called $x_k = z_k S H_k$ and $x'_k = z'_k S H_k$. When performing ML decoding or APP detection, we are interested in the equivalent Binary Shift Keying (BSK) modulation defined by the two points x_k and x'_k. The vector $(z_k - z'_k)S H_k$ has $s n_r$ circular symmetric Gaussian components. Thus, whatever the number of erroneous bits on a precoding time period, the obtained diversity is limited to $s n_r$. Having several erroneous bits per precoding time period is useless. On the contrary, if the erroneous bits are located on different precoding time periods and experience different fading random variables, a higher diversity is achieved. This is what we call the non-interference property. Furthermore, we will see in section 5 that an equi-distribution of erroneous bits on channel states is required to achieve a maximum coding gain. We call it the equi-distribution property. The ideal interleaver is defined as follows:

Definition 1 *(Ideal Interleaving) For any pair of codewords with w different bits at positions $i_1, \ldots, i_k, \ldots, i_w$, an ideal interleaver allocates the bits to transmitted symbols as follows:*

- *Non-interference property: $\forall i_k, i_{k'}$, bits at positions i_k and $i_{k'}$ are transmitted on different precoding time periods,*

- *Equi-distribution property: the bits at positions $i_1, \ldots, i_k, \ldots, i_w$ are as equiprobably distributed over all channel states as allowed by w.*

In practice, such an interleaver does not always exist. We will see in the following that the Singleton bound gives an existence condition of the ideal interleaver. In section 6, we present optimized interleavers that approach the ideal condition.

3.2 Exact pairwise error probability for ergodic channels without precoding

In [22], we have established a closed form expression for the conditional pairwise error probability on ergodic MIMO channels under ML decoding of the BICM and ideal channel interleaving. The mathematical derivation in this subsection follows [22]. Transmitted symbols are not precoded: $s = 1$, $S = I_{n_t}$ the $n_t \times n_t$ identity matrix. Thus, (2) reduces to $H_k = H_k^{[1]}$. Consider the pairwise error probability that a codeword $c \in \mathcal{C}$ is transmitted and a codeword $c' \in \mathcal{C}$ is decoded. The w different bits between the two codewords are transmitted in w different time periods, complementing one bit in the mapping of one of the n_t QAM symbols. The transmitted noiseless vectors corresponding to the two codewords (c, c') only differ in w positions. Let us define $Z = \{z_1, \ldots, z_w\}$ and $Z' = \{z_1', \ldots, z_w'\}$ the w-dimensional vectors corresponding to these positions and $X = \{x_1, \ldots, x_w\}$ and $X' = \{x_1', \ldots, x_w'\}$ the transmitted vectors corresponding to Z and Z'. We define $d_k = \|z_k - z_k'\|$. The Euclidean distance $\|X - X'\|$ depends on both the set of distances $\{d_1, \ldots, d_k, \ldots, d_w\}$ and the set of channel realizations $\mathcal{H}$. Let D denote the set of all Euclidean distances obtained by flipping one bit in the constellation Ω. Define the set $\Delta = \{\delta_1, \ldots, \delta_{n_d}\} \subset D$ with distinct elements from the sequence $(d_1, d_2, \ldots, d_w) \in \Delta^w \subset D^w$, i.e., the Euclidean distance d_k takes its values from the set Δ. Obviously, $n_d = |\Delta| \leq |D|$. Let the integer λ_k denote the frequency of δ_k in the sequence $(d_1, d_2, \ldots, d_w)$, $\sum_{n=1}^{n_d} \lambda_n = w$ and $\Lambda = \{\lambda_1, \ldots, \lambda_{n_d}\}$. The pairwise error probability conditioned on the channel realization set $\mathcal{H}$ and the Hamming weight w is expressed as

$$P_{w,\mathcal{H}}(c \to c') = P_{w,\mathcal{H}}(X \to X') = P\left(\sum_{k=1}^{w} LLR_k < 0\right) \tag{5}$$

where LLR_k is the k-th LLR, corresponding to the k-th error position, and is equal to

$$LLR_k = \frac{\|y_k - x_k'\|^2 - \|y_k - x_k\|^2}{2N_0} \sim \mathcal{N}\left(\frac{R_k}{2N_0}, \frac{R_k}{N_0}\right) \tag{6}$$

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$R_k = \|(z_k - z'_k)H_k\|^2$ has a chi-square distribution of order $2n_r$. Averaging over $\mathcal{H}$, we calculate the characteristic function of $E_{\mathcal{H}}\left[\sum_{k=1}^{w} LLR_k\right]$:

$$\psi(j\nu) = \left(\prod_{k=1}^{w}\left(\frac{-d_k^2}{2N_0}\right)^{-n_r}\right)\left(\prod_{n=-n_d, n\neq 0}^{n_d}[j\nu + \beta_n]^{-n_r\lambda_{|n|}}\right) \tag{7}$$

where

$$\begin{cases} n > 0, & \beta_n = \frac{1}{2}\left(1 + \sqrt{1 + \frac{8N_0}{\delta_n^2}}\right) \\ n < 0, & \beta_n = \frac{1}{2}\left(1 - \sqrt{1 + \frac{8N_0}{\delta_n^2}}\right) \end{cases} \tag{8}$$

Applying a partial fraction expansion, we obtain the expression of the pairwise error probability:

$$P_w(X \to X') = P_w(\Delta, \Lambda) = \prod_{k=1}^{w}\left(-\frac{2N_0}{d_k^2}\right)^{n_r}\sum_{n=1}^{n_d}\sum_{i=1}^{n_r\lambda_n}\frac{\alpha_{n,i}}{\left(\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{8N_0}{\delta_n^2}}\right)^i} \tag{9}$$

where the coefficients $\alpha_{n,i}$ are given by an identification of the coefficients of two series expansions in ϵ as in [22].

We compute the asymptotic expression when the noise level is low. Indeed, the coding gain and diversity are measured for high signal-to-noise ratios, where the performance has a linear asymptote on logarithmic scales.

$$P_w(\Delta, \Lambda) \underset{N_0 \to 0}{\sim} \binom{2n_r w - 1}{n_r w}\prod_{k=1}^{w}\left(\frac{2N_0}{d_k^2}\right)^{n_r} = \binom{2n_r w - 1}{n_r w}\left(\frac{2N_0}{\mathcal{G}_{ergo}(\Delta, \Lambda)}\right)^{wn_r} \tag{10}$$

with $\binom{n}{k} = n!/(k!(n-k)!)$. The diversity associated with the considered pairs of Hamming weight w is the exponent of $2N_0$, equal to wn_r. We define the coding gain or coding advantage as the coefficient dividing $2N_0$, i.e.,

$$\mathcal{G}_{ergo}(\Delta, \Lambda) = \left(\prod_{k=1}^{w}d_k^2\right)^{1/w} \tag{11}$$

All sequences $(d_1, \ldots, d_w)$ corresponding to the same pair (Δ, Λ) yield the same pairwise error probability. By averaging over all possible pairs (c, c') or equivalently over all sets of distances D^w, we obtain $P_w = E_{D^w}\left[P_w(\Delta, \Lambda)\right]$, the conditional probability that an error event of Hamming

weight w occurs. From this pairwise error probability, it is easy to estimate the FER or BER of the BICM with ideal interleaving thanks to a classical union bound on the weight enumeration function of the error correcting code. Moreover, we may derive a design criterion of the BICM from the coding gain $\mathcal{G}_{ergo}(\Delta, \Lambda)$ expression. In the following, we derive the coding gain for block-fading channels and linear precoding in order to obtain the ML design criterion of the ST-BICM.

3.3 Exact pairwise error probability for MIMO block fading channels without precoding

We assume that Definition 1 is satisfied. For a block-fading channel with n_c independent realizations in a frame, the decision variable between X and X' is still given by (5). However, the involved channel matrices are not independent as for an ergodic channel. The conditions of independence are the following:

- If two LLR random variables depend on two different channel realizations, they are independent.

- If two LLR random variables depend on the same channel realization but on different transmit antennas, the random variables are independent.

The maximum number of independent LLR variables is $n_c n_t$, the transmit diversity order. We choose the error correcting code so that $w \geq n_t n_c$. We now group the w random variables LLR into $\min(n_t n_c, w) = n_t n_c$ independent blocks. Let $LLR_{k,l,i}$ be the i-th log-likelihood ratio corresponding to the BSK transmission on the l-th antenna of the k-th block, $k = 1 \ldots n_c$, $l = 1 \ldots n_t$ and $i = 1 \ldots \kappa_{k,l}$, where $\kappa_{k,l}$ is the number of bits transmitted on the l-th antenna of the k-th block. We have $\sum_{k=1}^{n_c} \sum_{l=1}^{n_t} \kappa_{k,l} = w$. Finally, LLR is the sum of the $n_t n_c$ independent random variables $LLR_{k,l} = \sum_{i=1}^{\kappa_{k,l}} LLR_{k,l,i}$. Let $d_{k,l,i}$ denote the distance associated with $LLR_{k,l,i}$, and define $\gamma_{k,l}^2 = \sum_{i=1}^{\kappa_{k,l}} d_{k,l,i}^2$ the distance associated with $LLR_{k,l}$. We have

$$LLR_{k,l} \sim \mathcal{N}\left(\frac{R_{k,l}}{2N_0}, \frac{R_{k,l}}{N_0}\right) \tag{12}$$

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where $R_{k,l} = \gamma_{k,l}^2 \|H_k(l)\|^2$ and $H_k(l)$ is the l-th row of H_k. For all i, $LLR_{k,l,i}$ are transmitted over the equivalent $1 \times n_r$ SIMO channel defined by $H_k(l)$, which is chi-square distributed with degree $2n_r$. The $LLR_{k,l}$ variables are transmitted on independent channel states, as for the ergodic channel case, we directly apply (9) and obtain the conditional pairwise error probability closed-form expression

$$P_w(X \to X') = P_w(\Delta, \Lambda) = \prod_{k=1}^{n_c} \prod_{l=1}^{n_t} \left(-\frac{2N_0}{\gamma_{k,l}^2} \right)^{n_r} \sum_{n=1}^{n_d} \sum_{i=1}^{n_r \lambda_n} \frac{\alpha_{n,i}}{\left(\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{8N_0}{\delta_n^2}} \right)^i} \qquad (13)$$

where $\delta_n \in \Delta$ and (Δ, Λ) is the pair of sets representing the sequence $(\gamma_{1,1}, \ldots, \gamma_{n_t, n_c})$. The $\alpha_{n,i}$ coefficients are computed as for (9).

The asymptotic expression of $P_w(\Delta, \Lambda)$ is

$$P_w(\Delta, \Lambda) \underset{N_0 \to 0}{\sim} \binom{2n_r n_t n_c - 1}{n_r n_t n_c} \prod_{k=1}^{n_c} \prod_{l=1}^{n_t} \left(\frac{2N_0}{\gamma_{k,l}^2} \right)^{n_r} \qquad (14)$$

The diversity associated with the considered pairs of Hamming weight w is then equal to the exponent $n_t n_c n_r$. The coding gain is given by the geometrical mean of the $\gamma_{k,l}^2$ and is equal to

$$\mathcal{G}_{bf}(\Delta, \Lambda) = \left(\prod_{k=1}^{n_c} \prod_{l=1}^{n_t} \sum_{i=1}^{\kappa_{k,l}} d_{k,l,i}^2 \right)^{1/(n_t n_c)} \qquad (15)$$

We will see in the following how to use this coding gain as a design criterion for the ST-BICM optimization. We now consider an equivalent computation of the coding gain for a linearly precoded ST-BICM.

3.4 Exact pairwise error probability for MIMO block fading channels with precoding

When a linear precoder S of size $N_t \times N_t$ is used, the detector computes soft outputs on the N_t transmitted symbols using the equivalent channel matrix SH_k of size $N_t \times N_r$. The structure of

H_k is described in (2). SH_k can be seen as a correlated MIMO channel [42]. Under the ideal interleaving condition, we consider at most a single erroneous bit per block of s time periods in position $1 \le \ell \le mN_t$ inside the binary mapping of the transmitted symbol z, leading to symbol $\bar{z}^\ell$. For simplicity reasons, we assume that the error weight w satisfies $w \ge N_t N_c$. Moreover, we assume that the mapping is mono-dimensional: the BSKs are transmitted on a single selected input of the matrix SH_k. Let $LLR_{k,l,i}$ be the i-th variable among $\kappa_{k,l}$, corresponding to the transmission of a BSK on the equivalent $1 \times N_r$ channel $S_l H_k$, where S_l corresponds to the l-th row of S. We have $\sum_{k=1}^{N_c} \sum_{l=1}^{N_t} \kappa_{k,l} = w$. Let $d_{k,l,i}$ denote the BSK distance associated with $LLR_{k,l,i}$. We can use the factorization $LLR_{k,l} = \sum_{i=1}^{\kappa_{k,l}} LLR_{k,l,i}$ of all the LLR variables filtered with $S_l H_k$:

$$LLR_{k,l} \sim \mathcal{N}\left(\frac{R_{k,l}}{2N_0}, \frac{R_{k,l}}{N_0} \right) \tag{16}$$

where $R_{k,l} = \|V_{k,l} H_k\|^2$, $V_{k,l} = \gamma_{k,l} S_l$ and $\gamma_{k,l}^2 = \sum_{i=1}^{\kappa_{k,l}} d_{k,l,i}^2$. The variable $R_{k,l}$ is a generalized chi-square random variable with $2N_r$ correlated centered Gaussian components. The random variable $LLR_k = \sum_{l=1}^{N_t} LLR_{k,l}$ satisfies

$$LLR_k \sim \mathcal{N}\left(\frac{\sum_{l=1}^{N_t} R_{k,l}}{2N_0}, \frac{\sum_{l=1}^{N_t} R_{k,l}}{N_0} \right) \tag{17}$$

From appendix A, we get the following characteristic function:

$$E_{H_k}\left[\Psi_{LLR_k}(j\nu) \right] = \prod_{t=1}^{n_s} \prod_{u=1}^{n_t} \left(1 - \frac{\nu(j-\nu)}{2N_0} \vartheta_{k,u}^{[t]} \right)^{-n_r} \tag{18}$$

where $\vartheta_{k,u}^{[t]}$ is the u-th eigenvalue of

$$\Sigma_k^{[t]} = \sum_{l=1}^{N_t} \gamma_{k,l}^2 S'^{[t]*}_l S'^{[t]}_l = \sum_{l=1}^{N_t} \gamma_{k,l}^2 \sum_{i=1}^{s/n_s} S_l^{[t][i]*} S_l^{[t][i]} = M_k^{[t]*} M_k^{[t]} \tag{19}$$

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$M_k^{[t]}$ is as an $n_t \times n_t$ Hermitian square root matrix of $\Sigma_k^{[t]}$ and row vectors $S_l^{[t][i]}$ of size n_t and $s/n_s \times n_t$ matrices $S'_l^{[t]}$ are defined from S as follows:

$$
S = \begin{array}{c} \overbrace{S_l^{[1]}(1 \times N_t/n_s)}^{} \qquad\qquad\qquad \overbrace{S_l^{[n_s]}}^{} \\ \left[\begin{array}{cccccccc} S_1^{[1][1]} & \cdots & S_1^{[1][s/n_s]} & \cdots & S_1^{[n_s][1]} & \cdots & S_1^{[n_s][s/n_s]} \\ S_2^{[1][1]} & \cdots & S_2^{[1][s/n_s]} & & S_2^{[n_s][1]} & \cdots & S_2^{[n_s][s/n_s]} \\ \vdots & & & & \vdots & & \\ S_{N_t}^{[1][1]} & \cdots & S_{N_t}^{[1][s/n_s]} & \cdots & S_{N_t}^{[n_s][1]} & \cdots & S_{N_t}^{[n_s][s/n_s]} \end{array} \right] \\ \underbrace{\qquad}_{} \quad \underbrace{\qquad}_{} \qquad\qquad \underbrace{\qquad}_{} \quad \underbrace{\qquad}_{} \end{array}
\tag{20}
$$

$$N_t/s = n_t \text{ coefficients}$$

and

$$
S'_l^{[t]} = \begin{bmatrix} S_l^{[t][1]} \\ S_l^{[t][2]} \\ \vdots \\ S_l^{[t][s/n_s]} \end{bmatrix}
\tag{21}
$$

The set of eigenvalues $\vartheta_{k,u}^{[t]}$ is a function of the precoding matrix S and the BSK distances set D^w. Thanks to the independence of channel realizations for different k values, we can multiply the characteristic functions:

$$
\Psi(j\nu) = \prod_{k=1}^{N_c} \prod_{t=1}^{n_s} \prod_{u=1}^{n_t} \left(1 - \frac{\nu(j-\nu)}{2N_0} \vartheta_{k,u}^{[t]} \right)^{-n_r}
\tag{22}
$$

Denote $\Delta = \{\delta_v\}$ the set of n_δ square-roots of non-null eigenvalues extracted from the sequence defined by the $\vartheta_{k,u}^{[t]}$ values. Each eigenvalue δ_v^2 is repeated λ_v times. Observe that $n_\delta \le n_c n_t$. Finally, using the partial fraction expansion of $\Psi(j\nu)$ as for (9), we obtain the exact pairwise error probability $P_w(\Delta, \Lambda)$ conditioned on $d_H(c,c') = w$:

$$
P_w(\Delta, \Lambda) = \prod_{v=1}^{n_\delta} \left(-\frac{2N_0}{\delta_v^2} \right)^{\lambda_v n_r} \sum_{v=1}^{n_\delta} \sum_{i=1}^{n_r \lambda_v} \frac{\alpha_{v,i}}{\left(\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{8N_0}{\delta_v^2}} \right)^i}
\tag{23}
$$

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The asymptotic expression of $P_w(\Delta, \Lambda)$ is

$$P_w(\Delta, \Lambda) \underset{N_0 \to 0}{\sim} \binom{2n_r N_\delta - 1}{n_r N_\delta} \prod_{v=1}^{n_\delta} \left(\frac{2N_0}{\delta_v^2} \right)^{\lambda_v n_r} \tag{24}$$

where $N_\delta = \sum_{v=1}^{n_\delta} \lambda_v$ is the total number of non-null eigenvalues.

The diversity associated with the considered pairs of Hamming weight w is the exponent equal to $\sum_{v=1}^{n_\delta} \lambda_v n_r = N_\delta n_r$. The coding gain is given by

$$\mathcal{G}_{s,n_s}(\Delta, \Lambda) = \left(\prod_{v=1}^{n_\delta} \delta_v^{2\lambda_v} \right)^{1/N_\delta} \tag{25}$$

We have derived for any signal-to-noise ratio an exact expression of the pairwise error probabilities of a BICM with linear precoding, which is useful for a tight BER and FER estimation. The asymptotic expression leads to the well-known rank and determinant criteria [39][19] for space-time code optimization over MIMO block-fading channels, where the considered space-time code is the whole BICM structure. As a remark, the asymptotic design criterion is usually derived by first upperbounding the $Q(x)$ function by $\exp(-x^2/2)/2$ and then averaging over the channel realizations. The obtained asymptotic expression has a multiplying coefficient different from $\binom{2n_r N_\delta - 1}{n_r N_\delta}$, which is inexact but provides the same design criterion.

Moreover, we notice that applying the Tarokh criterion [39] on the rank and determinant to the precoder alone does not lead to the whole BICM optimization. Quasi-optimal linear precoders will be designed to achieve full diversity and approach optimal coding gain in section 5.

3.5   Evaluation of the Frame Error Rate

For ergodic channels, the frame error rate is easily computed via a truncated union bound. Indeed, only error events with minimum Hamming distance impact the error rate for a high signal-to-noise ratio and the observed diversity is equal to $n_r d_{Hmin}$. For block-fading channels, the frame error rate computation is much more tricky since each pairwise error probability is supposed to have

the full-diversity order $n_c n_t n_r$. Due to the random nature of each eigenvalue in (25), it is difficult to know the impact of each distance configuration on the final FER.

However, one may assume that for a sufficiently high signal-to-noise ratio, the FER satisfies the following expression:

$$FER \simeq \sum_w A_w E_{(\Delta,\Lambda|w)} \left[P_w(\Delta,\Lambda) \right] \qquad (26)$$

where A_w is weighting the impact of pairwise error probabilities with Hamming weight w in the global error probability and the expectation on (Δ, Λ) is allowed by the interleaver random structure. Let us define $\mathcal{G}$ the global coding gain. Since each pairwise error probability is supposed to have full diversity, we write

$$FER \simeq \binom{2n_r n_t n_c - 1}{n_r n_t n_c} \left(\frac{2N_0}{\mathcal{G}} \right)^{n_r n_t n_c} \qquad (27)$$

and

$$\mathcal{G}^{-n_r n_t n_c} = \sum_w A_w E_{(\Delta,\Lambda|w)} \left[\mathcal{G}(\Delta,\Lambda)^{-n_r n_t n_c} \right] \qquad (28)$$

where $\mathcal{G}(\Delta, \Lambda)$ is the coding gain associated with one pair of codewords. We note that optimizing independently all pairwise error probabilities, which will be done in the following, enhances the global performance. Moreover, we observe that the number of receive antennas does not affect the coding gain of a single pairwise error probability. The effect of the receive diversity appears in the expression of the global coding gain (28). As $n_r n_t n_c$ grows, the smallest coding gains have more impact on the final performance. Asymptotically, if $n_r n_t n_c \to +\infty$, only the nearest neighbors in the Euclidean code have an influence on the FER, as for AWGN channels.

We will see in section 5.1 that the best coding gain is achieved when all eigenvalues $\vartheta_{k,u}^{[t]}$ are equal. In this ideal configuration, the coding gain is shown to be the same as with the same coded modulation transmitted on a $1 \times n_c n_t n_r$ quasi-static SIMO channel. Simulating this latter case is

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less complex: the performance curve is semi-analytically computed using a reference curve on an AWGN channel. Alternatively, performance may be obtained by computing the Tangential Sphere Bound for spherical modulations [26]. In the following, *ideal BICM* will refer to the performance of the ideal configuration, which will be drawn on simulation results. This lower bound has the advantage to take the modulation and error correcting code into account and will be useful to evaluate the optimality of both the linear precoder and the channel interleaver.

4   The Singleton bound with linear precoder

Definition 1 ensures that any pair of codewords benefits from a full diversity order. In this section, we derive a condition on the existence of a practical interleaver that could achieve the conditions of Definition 1. Let us first make the following assumption:

Assumption 1 *The detector perfectly converts the $N_t \times N_r$ correlated MIMO N_c-block-fading channel SH_k with QAM input into a $1 \times sn_r$ SIMO $n_t n_c/s$-block-fading channel with BSK input, assuming that s is a divisor of $n_t n_c$.*

We will present in section 5 linear precoders that satisfy Assumption 1. Under this condition, the detector collects an amount of diversity equal to sn_r. The full diversity $n_t n_c n_r$ is collected by the detector when $s = n_t n_c$, but unfortunately, the APP signal detection has an exponential complexity in s. On the other hand, the BICM channel decoder is also capable of collecting a large amount of diversity, but the latter is still limited by the Singleton bound [28][29][33]. Hence, the lowest complexity solution that reaches full diversity is to draw advantage of the whole channel code diversity and recover the remaining diversity by linear precoding. The best way to choose the spreading factor s is given by the Singleton bound described hereafter.

The studied ST-BICM is a serial concatenation of a rate R_C binary convolutional code C, an interleaver of size $N_C L_C$ bits, and a QAM mapper followed by the precoder as described in

section 2. When S is the identity matrix, the ST-BICM diversity order is upper-bounded by [29]:

$$\Upsilon \leq n_r \left(\lfloor n_c n_t (1 - R_c) \rfloor + 1 \right) \tag{29}$$

The maximal diversity given by the outage limit under a finite size QAM alphabet also achieves the above Singleton bound [24]. With a vanishing coding rate, i.e., $R_c \leq 1/(n_c n_t)$, it is possible to attain the overall system diversity order $n_r n_c n_t$ produced by the receive antennas, the transmit antennas and the distinct channel states. Unfortunately, this is unacceptable due to the vanishing transmitted information rate. Precoding is one means to achieve maximum diversity with a non-vanishing coding rate.

The integer $N_b = n_c n_t / s$ is the best diversity multiplication factor to be collected by $\mathcal{C}$. The length of a $\mathcal{C}$ codeword is $L_{\mathcal{C}} N_{\mathcal{C}}$ binary elements. Let us group $L_{\mathcal{C}} N_{\mathcal{C}} / N_b$ bits into one non-binary symbol creating a non-binary code $\mathcal{C}'$. Now, $\mathcal{C}'$ is a length-N_b code built on an alphabet of size $2^{L_{\mathcal{C}} N_{\mathcal{C}} / N_b}$. The Singleton bound on the minimum Hamming distance of the non-binary $\mathcal{C}'$ becomes $D_H \leq N_b - \lceil N_b R_{\mathcal{C}} \rceil + 1$. Multiplying the previous inequality with the Nakagami law order $s n_r$ yields the maximum achievable diversity order after decoding:

$$\Upsilon \leq s n_r \left\lfloor \frac{n_c n_t}{s} (1 - R_{\mathcal{C}}) + 1 \right\rfloor \tag{30}$$

Finally, since Υ is upper-bounded by the channel intrinsic diversity and the minimum Hamming distance d_{Hmin} of the binary code, we can write

$$\Upsilon \leq \min \left(s n_r \left\lfloor \frac{n_c n_t}{s} (1 - R_{\mathcal{C}}) + 1 \right\rfloor ; n_t n_c n_r ; s n_r d_{Hmin} \right) = \Upsilon_{max} \tag{31}$$

If d_{Hmin} is not a limiting factor (we choose $\mathcal{C}$ accordingly), we can select the value of s that leads to a modified Singleton bound greater than or equal to $n_t n_c n_r$.

Proposition 1 *Considering a BICM with a rate $R_{\mathcal{C}}$ binary error-correcting code on an $n_t \times n_r$*

MIMO channel with n_c distinct channel states per codeword, the spreading factor s of a linear precoder must be a divisor of $n_t n_c$ and must satisfy $s \geq R_\mathcal{C} n_c n_t$ in order to achieve the full diversity $n_t n_c n_r$ for any pair of codewords. In this case, the ideal interleaving conditions can be achieved with an optimized interleaver.

The smallest integer s_{opt} satisfying the above proposition minimizes the detector's complexity. If $R_\mathcal{C} > 1/2$, then $s_{opt} = n_c n_t$ which involves the highest complexity. If $R_\mathcal{C} \leq 1/(n_c n_t)$, linear precoding is not required.

Tables 1 and 2 show the diversity order derived from the Singleton bound versus s and n_t, for $n_c = 1$ and $n_c = 2$ respectively. The values in bold indicate full diversity configurations. For example, in Table 1, for $n_t = 4$, $s = 2$ is a better choice than $s = 4$ since it leads to an identical diversity order with a lower complexity.

5   Linear precoder optimization

Many studies have been published on space-time spreading matrices introducing some redundancy, well-known as space-time block codes. On one hand, some of them are decoded by a low-complexity ML decoder, but they sacrifice transmission data rate for the sake of high performance. Among them, the Alamouti scheme [1] is the most famous, but is only optimal for a 2×1 MIMO channel. The other designs allowing for low ML decoding complexity are based on an extension of the Alamouti principle (e.g., DSTTD [41]) but also sacrifice the data rate. On the other hand, full rate space-time codes have recently been proposed [4][11][12][13][14][18][34]. However, their optimization does not take into account their concatenation with an error correcting code. In this section, we describe a near-ideal solution for linear precoding in BICMs under iterative decoding process. Our strategy is to separate the coding step and the geometry properties in order to express some criteria allowing the construction of a space-time spreading matrix for given channel parameters n_t, n_r and n_c. The inclusion of rotations to enhance the BICM performance over single antenna channels has been proposed in [31]. Our solution uses this concept for designing a space-time code

including a powerful error correcting code.

When the channel is quasi-static or block-fading with parameter n_c, the diversity is upper bounded by $n_c n_t n_r$ which may be more limiting than $n_r d_{Hmin}$ (e.g., $n_t = 2$, $n_r = 1$, $n_c=1$). We introduce a new design criterion of space-time spreading matrices that guarantees a diversity proportional to the spreading factor, within the upper-bound, and a maximal coding gain at the last iteration of an iterative joint detection and decoding.

5.1 Coding gain under both ideal interleaving and precoding

First we look for the best achievable coding gain for the fixed parameters n_t, n_r, n_c, $R_\mathcal{C}$ and the appropriate way to choose the error correcting code, the binary mapping, the linear precoder and its parameters s and n_s to achieve the ideal coding gain.

We want to achieve full diversity under ML decoding or iterative joint detection and decoding, this induces that there are $n_c n_t$ non-null eigenvalues $\vartheta_{k,u}^{[t]}$ (see (24)):

$$\mathcal{G}_{s,n_s}(\Delta, \Lambda) = \left(\prod_{k=1}^{N_c} \prod_{t=1}^{n_s} \prod_{u=1}^{n_t} \vartheta_{k,u}^{[t]} \right)^{1/(n_c n_t)} \tag{32}$$

Furthermore, we want to maximize the $\mathcal{G}_{s,n_s}(\Delta, \Lambda)$ expression. Assuming that each row S_l is normalized to 1, we get

$$\sum_{k=1}^{N_c} \sum_{t=1}^{n_s} \sum_{u=1}^{n_t} \vartheta_{k,u}^{[t]} = \sum_{k=1}^{N_c} \sum_{l=1}^{N_t} \gamma_{k,l}^2 = \sum_{k=1}^{N_c} \sum_{l=1}^{N_t} \sum_{i=1}^{\kappa_{k,l}} d_{k,l,i}^2 \tag{33}$$

Under this condition, the ideal coding gain is achieved when all eigenvalues are equal

$$\vartheta_{k,u}^{[t]} = \sum_{k'=1}^{N_c} \sum_{l=1}^{N_t} \frac{\gamma_{k',l}^2}{n_t n_c} \quad \forall (k,t,u) \tag{34}$$

which leads to

$$\mathcal{G}_{ideal}(\Delta, \Lambda) = \sum_{k=1}^{N_c} \sum_{l=1}^{N_t} \frac{\gamma_{k,l}^2}{n_t n_c} = \sum_{i=1}^{w} \frac{d_i^2}{n_t n_c} \qquad (35)$$

The exact pairwise error probability expression simplifies to the classical expression of the performance of a BPSK with distance $\sum_{j=1}^{w} d_j^2$ over a diversity channel with order $n_c n_t n_r$ [36]:

$$P_{w,ideal}(\Delta, \Lambda) = \left(1 - \left(1 + \frac{8N_0 n_t n_c}{\sum_{j=1}^{w} d_j^2}\right)^{-1/2}\right)^{n_c n_t n_r} \sum_{k=0}^{n_c n_t n_r - 1} \frac{\binom{n_c n_t n_r + k - 1}{k}}{2^{n_c n_t n_r + k}} \left(1 + \left(1 + \frac{8N_0 n_t n_c}{\sum_{j=1}^{w} d_j^2}\right)^{-1/2}\right)^{k}$$

$$(36)$$

As stated in the introduction, in an ST-BICM, precoded modulation symbols quantify the Shannon sphere and best quantization is obtained by uniformly distribute them on the sphere. After transmission on a fading channel, vectors belong to an ellipsoid obtained by applying an homothety on the sphere. From (34) and (35), we see that the ideal coding gain is obtained by equally distributing the Euclidean distance between two codewords among the $n_t n_c$ channel states. Hence, the Euclidean distance varies as a $n_t n_c n_r$ Nakagami distribution, according to the square norm of the ellipsoid axes. Thus, an ideal ST-BICM aims at uniformly distributing the precoded modulation symbols, whatever the channel realization, i.e., whatever the homothety. The ideal coding gain is a fundamental limit which cannot be outperformed. It is useful to evaluate how optimal the practical design of a BICM is. We aim at finding the best design, corresponding to eigenvalues which are as close to each other as possible. The more different from each other the eigenvalues are, the lower the product in (32) and the coding gain are. From (35), we see that the ideal coding gain is the same as for the same coded modulation transmitted on a $1 \times n_c n_t n_r$ single-input multiple-output (SIMO) channel, applying the appropriate E_b/N_0 normalization.

Without linear precoding, the ideal coding gain is only achieved if all $\gamma_{k,l}$ are equal. Remember that each $\gamma_{k,l}$ is a sum of $\kappa_{k,l}$ distances $d_{k,l,i}$. Thanks to the second point in Definition 1, the $\kappa_{k,l}$ values are close to $w/(n_t n_c)$ and their variance decreases when w increases. Thus, with a powerful error correcting code having minimum Hamming distance much greater than $n_t n_c$ and $|D|$, each $\gamma_{k,l}$ value is almost equal to the average $\sum_{k=1}^{N_c} \sum_{l=1}^{N_t} \sum_{i=1}^{\kappa_{k,l}} d_{k,l,i}^2/(N_t N_c)$ of $d_{k,l,i}$ values

and quasi-ideal coding gain is observed.

If the error correcting code is not powerful enough to achieve the ideal coding gain, i.e., the $\gamma_{k,l}$ values are very different, the linear precoder provides an additional coding gain by averaging the $\gamma_{k,l}$ values, as we will see in the following. First, we derive the optimal coding gain which can be achieved using an ideal linear precoder for a given binary labeling and error correcting code. Variables $\gamma_{k,l}$ for different k values correspond to independent channel realizations H_k which are not linked by the linear precoder. Thus, random variables $\prod_{t=1}^{n_s} \prod_{u=1}^{n_t} \vartheta_{k,u}^{[t]}$ are independent for distinct values of k. The optimal coding gain with linear precoding is

$$\mathcal{G}_{s,n_s,opt}(\Delta, \Lambda) = \prod_{k=1}^{N_c} \left(\sum_{l=1}^{N_t} \frac{\gamma_{k,l}^2}{n_t n_s} \right)^{1/N_c} \tag{37}$$

Equation (37) means that an optimal linear precoder is capable of making eigenvalues equal for a same k. However, for different values of k, eigenvalues $\vartheta_{k,u}^{[t]}$ are different, which induces a coding gain loss. When the mapping and error correcting code are given and the interleaving is ideal, the choice of linear precoding parameters impacts on optimal coding gain. Let us consider codewords that are equidistant from the transmitted codeword, i.e., a set of distance configurations corresponding to a same value of $\sum_{k,l} \gamma_{k,l}^2$. The variance of $\sum_{l=1}^{N_t} \gamma_{k,l}^2/(n_t n_s)$ over this set decreases when n_s increases, as the number of distances building each $\gamma_{k,l}$ is higher. The lower the variance of eigenvalues, the higher the coding gain. Thus, $\mathcal{G}_{s,n_s,opt}(\Delta, \Lambda)$ is an increasing function of n_s and, for a given s, we should choose $n_s = \min(s, n_c)$. The optimal coding gain $\mathcal{G}_{s,\min(s,n_c),opt}(\Delta, \Lambda)$ is an increasing function of s. If $n_s = n_c$, which implies $s = n_t n_c$, the ideal coding gain is achieved by the optimal precoder. Finally, we can surround the coding gain at full diversity as follows:

$$\forall s \quad \mathcal{G}_{ideal}(\Delta, \Lambda) \geq \mathcal{G}_{s,\min(s,n_c),opt}(\Delta, \Lambda) \geq \mathcal{G}_{s,1,opt}(\Delta, \Lambda) \geq \mathcal{G}_{1,1,opt}(\Delta, \Lambda) \geq \mathcal{G}_{bf}(\Delta, \Lambda) \tag{38}$$

If, for any pairwise error probability, $\mathcal{G}_{bf}(\Delta, \Lambda) \simeq \mathcal{G}_{ideal}(\Delta, \Lambda)$, the linear precoder optimization is useless from a coding gain point-of-view. However, obtaining near-ideal coding gain without

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precoding requires an optimization of the error correcting code and mapping for any pairwise error probability, which is intractable for non-trivial modulations and codes. Furthermore, the first objective of linear precoding is the diversity control, which has a high influence on the performance even at medium FER ($10^{-2} \sim 10^{-3}$), especially for low diversity orders. Therefore, precoding is often useful in the BICM structure.

After the impact of the linear precoding for a given pairwise error probability, let us consider the behavior of the global performance under linear precoding. As stated in section 3.5, if $n_r n_t n_c$ grows, the pairs of codewords providing the smallest coding gains have more impact on the final performance. Since the linear precoder provides a more substantial gain for the low Hamming weight configurations, the coding gain of the linear precoder will be magnified as the diversity grows.

Example of ideal coding gain: In order to illustrate the role of the linear precoding in the coding gain optimization, we consider a 2×1 quasi-static MIMO channel and a pairwise error probability between two codewords separated by a Hamming distance of w bits. Fig. 3 represents the distribution of the two γ_1 and γ_2 values over the two transmit antennas without linear precoding. Bits transmitted on antennas 1 and 2 are transmitted on the sets of time periods T_1 and T_2, respectively. Thanks to ideal interleaving, $T_1 \bigcap T_2 = \emptyset$. This illustrates the factorization of the distances into the γ values. The instantaneous coding gain is equal to $\sqrt{\gamma_1^2 \gamma_2^2}$. Now let us consider a specific linear precoder, which spreads the values γ_1 and γ_2 as presented in Fig. 4 over two time periods and two transmit antennas dividing the squared distance in two equal parts $\gamma_1^2/2$ and $\gamma_2^2/2$ respectively. The average value $(\gamma_1^2 + \gamma_2^2)/2$ is transmitted on each antenna, the coding gain is optimal and equal to $(\gamma_1^2 + \gamma_2^2)/2$. For example, consider a BPSK modulation and a pairwise error probability with Hamming weight 3. With optimal linear precoding, the ideal interleaving provides for example $\gamma_1^2 = 2 \times 2^2$ and $\gamma_2^2 = 1 \times 2^2$. With optimal linear precoding, we have a distance $(2 \times 2^2 + 1 \times 2^2)/2$ associated with each antenna. The ratio between the two averaged coding gains

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is equal to $\sqrt{9/8}$, i.e., we expect a gain of 0.26 dB when using linear precoding. With $w = 5$ and $w = 11$, the coding gain becomes $10 \log_{10}(\sqrt{24/25}) \simeq 0.09$ dB and $10 \log_{10}(\sqrt{120/121}) \simeq 0.02$ dB, respectively. The higher the Hamming weight involved in the pairwise error probability is, the less the coding gain provided by linear precoding is. ∎

We see on Table 3 the best gain to be provided by linear precoding for a quasi-static channel with BPSK input with respect to a full diversity unprecoded scheme. These gains are particularly low because the error correcting code aims at recovering a large amount of coding gain. This illustrates that BICMs are very efficient transmission schemes. As a remark, if a modulation with higher spectral efficiency is used with Gray mapping, the nearest neighbor in the Euclidean code has the same distance configuration as if a BPSK modulation was used. Moreover, for high diversity orders, the global error rate for high E_b/N_0 will be dominated by the neighbors and the gain provided by linear precoding will be very close to the ones shown in Table 3. However, if the diversity is low, the gains provided by linear precoding may be much more important. Assume that a 16-QAM modulation with Gray mapping is transmitted on a $n_t = 2$ quasi-static channel. For instance, if $w = 5$, there exists a neighbor with distance configuration $(3A, 3A, 3A, A, A)$ (e.g., see [22]), and $\gamma_1^2 = 9A^2 + 9A^2 + 9A^2$, $\gamma_2^2 = A^2 + A^2$. The gain to be provided by linear precoding is equal to $10 \log_{10}(29/2/\sqrt{54}) = 2.95$ dB. As already stated, the final coding gain is equal to a weighted sum of all the coding gains, where the weighting coefficients cannot be easily computed in the case of low diversity orders.

Even if linear precoding does not always provide a substantial coding gain, its prior aim is the diversity order control. Thus, we will focus on the design of linear precoders aiming at reaching full diversity and maximizing the coding gain for any set of parameter (n_t, s, n_s).

5.2 A new class of linear precoders

Under linear precoding, the optimal coding gain is achieved if all $\vartheta_{k,u}^{[t]}$ variables are equal for a same k. Let us first consider the eigenvalues associated with the independent realizations in the spreading matrix, indexed by t. First, two matrices $M_k^{[t_1]}$ and $M_k^{[t_2]}$, as introduced in (19), should have the same eigenvalues, which is satisfied if $\forall (t_1, t_2), M_k^{[t_1]} = R^{t_1,t_2*} M_k^{[t_2]} R^{t_1,t_2}$, where R^{t_1,t_2} is a unitary matrix, for example a rotation. Hence, $\forall (t_1, t_2), S_l'^{[t_1]} = S_l'^{[t_2]} R^{t_1,t_2}$. The precoding sub-part $S_l^{[t_1]}$, with spreading factor $s' = s/n_s$, experiences a quasi-static channel. We assume that s' is an integer, divisor of n_t. It is sufficient to design the first sub-part of the precoder matrix rows for a quasi-static channel and rotate it to compute the other sub-parts. Furthermore, any choice of R^{t_1,t_2} leads to the same performance because the eigenvalues remain unchanged. The condition simplifies to $\|S_l'^{[t_1]}\| = \|S_l'^{[t_2]}\|$.

Let us now optimize for a given index t the equivalent precoder over the quasi-static channel $\mathrm{diag}\left(H_k^{[t]}, \ldots, H_k^{[t]}\right)$, in which $H_k^{[t]}$ is repeated s' times. If all the eigenvalues of $M_k^{[t]} M_k^{[t]*}$ are equal, $M_k^{[t]}$ and $M_k^{[t]*}$ are weighted unitary matrices and

$$M_k^{[t]} M_k^{[t]*} = M_k^{[t]*} M_k^{[t]} = \sum_{l=1}^{N_t} \gamma_{k,l}^2 \sum_{i=1}^{s'} S_l^{[t][i]*} S_l^{[t][i]} \tag{39}$$

Matrix $S_l^{[t][i]*} S_l^{[t][i]}$ has rank one and matrix $\sum_{i=1}^{s'} S_l^{[t][i]*} S_l^{[t][i]}$ has maximum rank s'. If $s' < n_t$, it can be shown that it is impossible to get all eigenvalues equal to $\sum_{l=1}^{N_t} \gamma_{k,l}^2 / n_t n_s$ as required to achieve the optimal coding gain. However, in order to insure that $M_k^{[t]} M_k^{[t]*}$ has a rank n_t and that the eigenvalues are as equal as possible, we group ss' values $\gamma_{k,l}$ together and associate them with one of the n_t/s' groups of s' eigenvalues: we denote $S_l^{[t][i][j]}$ the j-th sub-part of size s' of $S_l^{[t][i]}$ and $\{l_2, l_1\}$ the index of the $(l_2 - 1)ss' + l_1$-th row of S, where $l_2 \in [1, n_t/s'], l_1 \in [1, ss']$. Let us assume that $S_{\{l_2,l_1\}}^{[t][i]}$ has only one non-null sub-part in position l_2, i.e.,

$$\forall j \neq l_2 \quad S_{\{l_2,l_1\}}^{[t][i][j]} = [0, \ldots, 0] \tag{40}$$

Considering such a structure is equivalent to considering separate precoding on n_t/s' distinct groups of s' transmit antennas. We have

$$\sum_{l=1}^{N_t} \gamma_{k,l}^2 {S'}_l^{[t]*} {S'}_l^{[t]} = \sum_{l_2=1}^{n_t/s'} \sum_{l_1=1}^{ss'} \gamma_{k,\{l_2,l_1\}}^2 \sum_{i=1}^{s'} S_{\{l_2,l_1\}}^{[t][i]*} S_{\{l_2,l_1\}}^{[t][i]} \tag{41}$$

$$= \sum_{l_2=1}^{n_t/s'} \sum_{l_1=1}^{ss'} \gamma_{k,\{l_2,l_1\}}^2 \mathfrak{D}_{l_2} \left(\sum_{i=1}^{s'} S_{\{l_2,l_1\}}^{[t][i][l_2]*} S_{\{l_2,l_1\}}^{[t][i][l_2]} \right) \tag{42}$$

where $\mathfrak{D}_{l_2}(A)$ is a block diagonal matrix with only one non-null block A in position l_2. We choose $S_{\{l_2,l_1\}}^{[t][i][l_2]}$ proportional to the i-th row of a $s' \times s'$ unitary matrix, such as $\|S_{\{l_2,l_1\}}^{[t][i][l_2]}\|^2 = 1/s$:

$$\sum_{l=1}^{N_t} \gamma_{k,l}^2 {S'}_l^{[t]*} {S'}_l^{[t]} = \sum_{l_2=1}^{n_t/s'} \sum_{l_1=1}^{ss'} \gamma_{k,\{l_2,l_1\}}^2 \mathfrak{D}_{l_2} \left(\frac{1}{s} I_{s'} \right) \tag{43}$$

$$= \frac{1}{s} \sum_{l_1=1}^{ss'} \text{diag} \left(\gamma_{k,\{1,l_1\}}^2 I_{s'}, \ldots, \gamma_{k,\{n_t/s',l_1\}}^2 I_{s'} \right) \tag{44}$$

which leads to

$$l_2 \leq n_t/s', u \leq s', \quad \vartheta_{k,(l_2-1)ss'+u}^{[t]} = \frac{1}{s} \sum_{l_1=1}^{ss'} \gamma_{k,\{l_2,l_1\}}^2 \tag{45}$$

The random variables $\gamma_{k,\{l_2,l_1\}}^2$ are independent and identically distributed for different values of l_1 and l_2, the coding gain is

$$\mathcal{G}_{s,n_s}(\Delta, \Lambda) = \prod_{k=1}^{N_c} \prod_{l_2=1}^{n_t/s'} \left(\sum_{l_1=1}^{ss'} \frac{\gamma_{k,\{l_2,l_1\}}^2}{s} \right)^{s'/(N_c n_t)} \tag{46}$$

For any value of n_s, the gain expressed in (46) is a geometric mean of order $n_t n_c/s$. For a given realization $\{d_1, \ldots, d_w\}$, a given s and for any n_s, $\sum_{k=1}^{N_c} \sum_{l_2=1}^{n_t/s'} \sum_{l_1=1}^{ss'} \gamma_{k,\{l_2,l_1\}}^2$ and thus $\sum_{l_1=1}^{ss'} \gamma_{k,\{l_2,l_1\}}^2$ are constant, ensuring the same coding gain. However, such a precoder does not achieve the optimal coding gain for any value of s'. The summation is made over ss' different values whereas the optimal coding gain in (37) necessitates a summation over sn_t values. Only if ss' is high enough, the obtained coding gain is almost optimal. If $s' = n_t$, the complete spatial transmit diversity is collected by the detector and the optimal coding gain is achieved.

Proposition 2 *Dispersive Nucleo Algebraic (DNA) Precoder* Let S be the $N_t \times N_t$ precoding matrix of a BICM over a $n_t \times n_r$ MIMO n_c-block-fading channel. Assume that S precodes a channel block diagonal matrix with s blocks and n_s channel realizations. We denote s the spreading factor, $N_t = sn_t$ and $s' = s/n_s$. Let $S_l^{[t]}$ be the t-th sub-part of size N_t/n_s of the l-th row of S. Let $S_l^{[t][i]}$ be the i-th sub-part of size n_t of $S_l^{[t]}$. Let $S_l^{[t][i][j]}$ be the j-th sub-part of size s' of $S_l^{[t][i]}$. The sub-part $S_l^{[t][i][j]}$ is called nucleotide. The linear precoder guarantees full diversity and quasi-optimal coding gain at the decoder output under maximum likelihood decoding of the BICM if it satisfies the two conditions of null nucleotides and orthogonal nucleotides for all $t \in [1, n_s], i \in [1, s'], l_1 \in [1, ss'], l_2 \in [1, n_t/s']$ and $\{l_2, l_1\} = (l_2 - 1)ss' + l_1$:

$$
\begin{cases}
\forall j \neq l_2, j \in [1, n_t/s'], \quad S_{\{l_2,l_1\}}^{[t][i][j]} = 0_{1 \times s'} & \underline{\textit{Null Nucleotide condition}} \\
\\
\forall i' \neq i, i' \in [1, s'], \quad S_{\{l_2,l_1\}}^{[t][i][l_2]} S_{\{l_2,l_1\}}^{[t][i'][l_2]*} = \frac{1}{s}\mathfrak{d}(i - i') & \underline{\textit{Orthogonal Nucleotide condition}}
\end{cases}
\tag{47}
$$

where $\mathfrak{d}(0) = 1$ and $\mathfrak{d}(x \neq 0) = 0$.

Let us take for example $n_t = 4$, $n_s = 1$ and $s = 2$. A DNA matrix would have the following structure:

$$
\text{DNA}(n_t = 4, n_s = 1, s = 2) =
\begin{bmatrix}
S_{\{1,1\}}^{[1][1][1]} & 0 & S_{\{1,1\}}^{[1][2][1]} & 0 \\
S_{\{1,2\}}^{[1][1][1]} & 0 & S_{\{1,2\}}^{[1][2][1]} & 0 \\
S_{\{1,3\}}^{[1][1][1]} & 0 & S_{\{1,3\}}^{[1][2][1]} & 0 \\
S_{\{1,4\}}^{[1][1][1]} & 0 & S_{\{1,4\}}^{[1][2][1]} & 0 \\
0 & S_{\{2,1\}}^{[1][1][2]} & 0 & S_{\{2,1\}}^{[1][2][2]} \\
0 & S_{\{2,2\}}^{[1][1][2]} & 0 & S_{\{2,2\}}^{[1][2][2]} \\
0 & S_{\{2,3\}}^{[1][1][2]} & 0 & S_{\{2,3\}}^{[1][2][2]} \\
0 & S_{\{2,4\}}^{[1][1][2]} & 0 & S_{\{2,4\}}^{[1][2][2]}
\end{bmatrix}
\tag{48}
$$

Now, let us consider a linear precoder matrix S that satisfies Proposition 2. We build a $ss' \times N_r$ matrix $H_k^{\{i\}}$ from the rows of H_k corresponding to the i-th group of s' transmit antennas. Elements of $H_k^{\{i\}}$ are defined as follows:

$$\forall i = 0 \ldots n_t/s' - 1, \forall j = 0 \ldots s' - 1, \forall u = 0 \ldots s - 1, \forall v = 0 \ldots N_r - 1,$$

$$H_k^{\{i\}}(j + us', v) = H_k(j + un_t + is', v) \tag{49}$$

Likewise, $S^{\{i\}}$ is the $ss' \times ss'$ matrix obtained from the i-th block of ss' rows of S and every n_t/s'-th block of s' columns beginning with the i-th block. We easily show that

$$SH_k = \begin{bmatrix} S^{\{1\}} H_k^{\{1\}} \\ S^{\{2\}} H_k^{\{2\}} \\ \vdots \\ S^{\{n_t/s'\}} H_k^{\{n_t/s'\}} \end{bmatrix} \tag{50}$$

which means that the matrix S independently precodes the n_t/s' groups of transmit antennas.

Thus, the optimization may be split into n_t/s' independent optimizations of linear precoders for $s' \times n_r$ MIMO n_s-block-fading channels with linear spreading factor s. As $s = s'n_s$, full space-time spreading of the $s' \times n_r$ block-fading channel is performed, i.e., the maximum diversity order sn_r is collected by the detector.

From (15) and (46), we notice that, at the decoder input and under ideal interleaving condition, the linear precoder at the transmitter end and the detector at the receiver end allow the conversion of the $n_t \times n_r$ MIMO channel with n_c independent blocks into a $1 \times sn_r$ SIMO channel with $n_c n_t/s$ independent blocks with BSK input. The independence of the blocks is provided by the structure of the linear precoding matrix:

1. The null nucleotides dispatch the transmitted symbols on n_t/s' different blocks of s' antennas.

2. The orthogonal nucleotides provide full diversity and a coding gain increasing with the

spreading factor.

For instance, if a rate $1/2$ BICM is transmitted on a quasi-static 4×2 MIMO channel, linear precoding with $s = 2$ is required to achieve full diversity: a full-rate space-time block code with spreading factor $s = s' = 2$ may independently be applied on 2 separate groups of 2 transmit antennas. Good 2×2 space-time block codes are for instance the TAST [12] and the Golden code [4].

Assume that $n_c = 1$, $n_t = n_r = 2$ and $s = s' = 2$. The Golden code is the best space-time code for uncoded 2×2 quasi-static MIMO channels. However, it does not satisfy the equal norm property of orthogonal nucleotides in Proposition 2. Indeed, one row of the Golden linear precoder contains two non-null coefficients of square norm $\alpha_1 = 0.277$ and $\alpha_2 = 0.723$, respectively. Thus (46), which assumes equality between the eigenvalues of $M_k^{[t]} M_k^{[t]*}$, does not hold. It can be shown that (let $\gamma_i^2 = \gamma_{1,\{1,i\}}^2$)

$$\mathcal{G}_{Golden}(\Delta, \Lambda) = \sqrt{\left(\alpha_1 \left(\gamma_1^2 + \gamma_4^2\right) + \alpha_2 \left(\gamma_2^2 + \gamma_3^2\right)\right) \left(\alpha_1 \left(\gamma_2^2 + \gamma_3^2\right) + \alpha_2 \left(\gamma_1^2 + \gamma_4^2\right)\right)} \qquad (51)$$

where $\gamma_i^2 = \gamma_{1,\{1,i\}}^2$. As d_{Hmin} increases, $(\gamma_1^2 + \gamma_4^2)/(\gamma_2^2 + \gamma_3^2)$ tends to 1 for any pairwise error probability and $\mathcal{G}_{Golden}(\Delta, \Lambda) \to \mathcal{G}_{2,1,opt}(\Delta, \Lambda)$: The error correcting code limits the coding loss due to the non-equal norm of the sub-parts of the Golden code. As a remark, if $\gamma_2^2 + \gamma_3^2 = 0$, which is the worst case, the coding loss is $10 \log_{10}(\sqrt{\alpha_1 \alpha_2/4}) = 0.5$ dB.

With DNA precoder and ideal interleaving, Assumption 1 is satisfied and the modified Singleton bound on the diversity order can apply. All results from the field of error correction coding over block-fading channels directly apply without any modification to the new $1 \times sn_r$ SIMO channel with $n_t n_c/s$ independent blocks.

5.3 The genie method design criterion for full spreading linear pre-coders $(s' = n_t)$

A linear precoding design criterion based on the genie performance optimization at the detector output has been proposed in [8]. When a genie gives a perfect information feedback on the mn_t coded bits required in the APP detector computation, the performance is computed by averaging all the pairwise error probabilities obtained when changing only one bit out of mn_t. Denote d the distance of the BSK. Assume that the BSK is transmitted on antenna l, the asymptotic expression of the error probability with genie is

$$P_{genie}(\Delta, \Lambda) \underset{N_0 \to 0}{\sim} \binom{2n_r N_\delta - 1}{n_r N_\delta} \prod_{v=1}^{n_\delta} \left(\frac{\delta_v^2}{2N_0} \right)^{-n_r \lambda_v} \tag{52}$$

where $\Delta = \{\delta_v\}$ is the set of square-roots of distinct non-null eigenvalues of $d^2 S_l'^{[t]*} S_l'^{[t]}$ for all t, λ_v their frequency and N_δ their number. In the best case, there are s non-null eigenvalues and the coding gain is maximized if they are equal. First, a sufficient condition to have an equality between the eigenvalues of $S_l'^{[t_1]*} S_l'^{[t_1]}$ and $S_l'^{[t_2]*} S_l'^{[t_2]}$ is $\|S_l'^{[t_1]}\|^2 = \|S_l'^{[t_2]}\|^2$. Then, all eigenvalues of $S_l'^{[t]*} S_l'^{[t]}$ are equal if $S_l'^{[t]}$ is a unitary matrix, which leads to the following proposition:

Proposition 3 *A linear precoder achieving a diversity order sn_r with maximum coding gain at the detector output must satisfy the following conditions under perfect iterative APP decoding of the space-time BICM:*

1. *The n_s subparts of the rows in the $sn_t \times sn_t$ precoding matrix have the same Euclidean norm*

2. *In each of the n_s subparts, the s subparts (nucleotides) are orthogonal and have the same Euclidean norm*

Proposition 3, which is more intuitive, is equivalent to Proposition 2 only if $s' = n_t$, i.e., in case of full spreading.

5.4 Non-full spreading quasi-optimal linear precoder: DNA cyclotomics

If $s' \neq n_t$, Proposition 3 is not optimal in terms of maximum likelihood performance. However, we can split the optimization of a $N_t \times N_t$ linear precoder with spreading factor s into n_t/s' optimizations of full spreading $N'_t \times N'_t$ linear precoders with $N'_t = s's$. The optimization of S is now done in two steps:

1. Apply the genie method to design a full spreading $N'_t \times N'_t$ linear precoder for a $s' \times n_r$ MIMO channel with n_s blocks, satisfying Proposition 3,

2. Place the non-null sub-parts in S as described in Proposition 2.

Cyclotomic rotations [6] provide good performance on ergodic Rayleigh SISO channels and have the great advantage to exist for any number of complex dimensions. Moreover, any coefficient has a unity norm which implies that the norm condition of Proposition 3 is naturaly satisfied. We modified the cyclotomic matrices to satisfy the orthogonality condition in the case of full spreading $s = n_t n_s$. The coefficients of S are equal to

$$
S_{l,v+(i-1)n_t+(t-1)n_t^2} =
$$
$$
\frac{1}{\sqrt{N_t}} \exp\left(2j\pi \left[(l-1)\left(\frac{1}{\Phi^{-1}(2N_t)} + \frac{(t-1)n_t^2+(i-1)n_t+v-1}{N_t}\right) + (i-1)\left(\frac{1}{\Phi^{-1}(2n_t)} + \frac{v-1}{n_t}\right)\right]\right) \tag{53}
$$

We denote $\mathcal{S}(n_t, n_s, n_t n_s)$ the modified cyclotomic rotation designed for a $n_t \times n_r$ MIMO block-fading channel, assuming that the precoder experiences n_s channel realizations. The last parameter in $\mathcal{S}(n_t, n_s, n_t n_s)$ denotes the spreading factor.

To satisfy Proposition 2, which gives the design criterion for non-full spreading quasi-optimal linear precoders, we follow the two steps described above. Following (53), we first construct $\mathcal{S}(s', n_s, s)$ designed for full spreading of a $s' \times n_r$ MIMO block-fading channel with n_s channel states in each precoded matrix. Then, we place n_t/s' times each subpart of $\mathcal{S}(s', n_s, s)$ in the precoding matrix in order to satisfy Proposition 2 and construct the quasi-optimal linear precoder

$\mathcal{S}(n_t, n_s, s)$ for any set of parameters n_t, n_s and s. Its coefficients are equal to

$$\forall l_2 \in [1, n_t/s'], \forall l_1 \in [1, ss'], \forall t \in [1, n_s], \forall i \in [1, s'], \forall v \in [1, s'],$$

$$S_{(l_2-1)s's+l_1,v+(l_2-1)n_t/s'+(i-1)n_t+(t-1)s'n_t} =$$

$$\frac{1}{\sqrt{N_t'}} \exp\left(2j\pi\left[(l_1-1)\left(\frac{1}{\Phi^{-1}(2N_t')} + \frac{v-1+(i-1)s'+(t-1)s'^2}{N_t'}\right) + (i-1)\left(\frac{1}{\Phi^{-1}(2s')} + \frac{v-1}{s'}\right)\right]\right) \quad (54)$$

and 0 elsewhere.

5.5   Performance of the quasi-optimal precoder with iterative receiver

We have presented quasi-optimal linear precoders providing good coding gain and full diversity ML performance under ideal interleaving. However, the ML decoder of the global Euclidean code does not exist and we process iterative joint detection and decoding. Proposition 2 is satisfied by an infinity of matrices, all providing the same ML performance. Let us consider the performance behavior after the first iteration. As no a priori information is available at the detector, errors before decoding are numerous and not necessarily transmitted on different precoding time periods. Let us consider one precoding time period and assume that we observe two erroneous bits. If the bits are transmitted on the same modulation symbol, the Euclidean distance d_k changes but this does not affect the linear precoder optimization. However, if the two bits are placed onto two different rows of S, the average performance might be modified and interference inside a block and between blocks should be considered. An optimization of the precoder following the Tarokh criterion should be done, under the conditions presented in Proposition 2. Simulation results show that the modified cyclotomic rotation has good uncoded ML performance, close to algebraic full rate space-time block codes. Thus, we expect good performance at the first iteration of a joint detection and decoding process, which is desirable to reduce the number of iterations needed to achieve the near ML performance and to provide good performance with non-iterative receivers. The optimization of the first iteration is not addressed in this paper, but first answers are given in [30].

6 Practical interleaver design for convolutional codes

The maximum diversity to be gathered is limited by the characteristics of the channel, the linear precoding spreading factor and the minimum Hamming distance of the binary code, all summarized in (31). Assume that the linear precoder spreading factor s is chosen such that diversity order is maximized, $\Upsilon_{max} = n_c n_t n_r$. Thus, there exists an interleaver that allows ML performance with full diversity. We present a new BICM interleaver design which satisfies Definition 1 and leads to the concept of full diversity BICM since the system exhibits a predetermined diversity whatever the parameters of the considered block-fading channel.

We first build an interleaver that enables to achieve maximum diversity on an $n_t \times n_r$ quasi-static MIMO channel ($n_c = 1$) with BPSK input. Then, we generalize the interleaver construction to apply it to higher spectral efficiency modulations, linear precoding and finally block-fading channels ($n_c > 1$).

6.1 Interleaver design for quasi-static MIMO channels with BPSK input

On quasi-static channels, a codeword undergoes only one channel realization. Let us consider an error event in the code trellis for which w coded bits differ from the transmitted codeword. As all error events are supposed to have a non-zero probability, the interleaver should be designed for any of them. Let us ensure the equi-distribution property that $L_I N_{\mathcal{C}}$ successive coded bits, $L_I N_{\mathcal{C}}$ being the length of an error path with L_I branches, are transmitted by all the n_t transmit antennas in the same proportion. To optimize performance, we must also ensure the non-interference of erroneous bits within the same time period. In the ML sense, two interfering erroneous bits may either degrade the diversity or the coding gain. When considering the graph representation of our system model in Fig. 1, a time period corresponds to one channel node. Probabilistic messages on bits should be independent. Practically, bits inside a channel node should be connected to distant positions in the code trellis. These conditions lead to a design criterion for quasi-static channels, well known in the algebraic space-time coding theory as the *rank criterion* [39] and applied here

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to the BICM interleaver.

To design an interleaver with size $L_{\mathcal{C}}N_{\mathcal{C}}$ ensuring that consecutive bits are mapped on different symbol time periods over all the transmit antennas, we demultiplex the $L_{\mathcal{C}}N_{\mathcal{C}}$ coded bits into n_t vectors of length $L_{\mathcal{C}}N_{\mathcal{C}}/n_t$. Each of these n_t sub-frames is separately interleaved and transmitted on a predetermined transmit antenna. However, the demultiplexing step is not simply processed via the periodical selection of every n_t bits. Indeed, let us observe the example in Fig. 5. Error positions are not equally distributed modulo $n_t = 2$. For any convolutional code, we apply the following demultiplexing in order to equally distribute erroneous bits on the n_t transmit antennas

$$0 \leq i < n_t, \ 0 \leq j < L_{\mathcal{C}}N_{\mathcal{C}}/n_t, \qquad V_i(j) = V\left((i+j) \bmod n_t + jn_t\right) \tag{55}$$

where V is the codeword to be demultiplexed, V_i is the i-th demultiplexed frame. This ensures the uniform distribution of erroneous bits over n_t transmit antennas all along the transmitted frame. Let us now limit the interference of erroneous bits during the same time period. We assume that only simple error events occur. If the same interleaver is used for all sub-frames, n_t consecutive bits are in the same position of interleaved sub-frames and we can limit the interference by sliding each sub-frame by one bit position and transmit all frames serially on their associated antennas. Yet, this does not guarantee that $L_I N_{\mathcal{C}}$ successive bits are transmitted over distinct time periods. To satisfy this strong condition, we use a particular S-random interleaver [16] with a sliding input separation which guarantees that any L_I successive bits in the interleaved frames are not transmitted during the same block of n_t time periods. If we consider that bit position i is placed at position $\Pi_s(i)$ by the interleaver Π_s, we should have

$$0 \leq j < L_{\mathcal{C}}N_{\mathcal{C}}/n_t - L_I, \ \ 0 \leq i < L_I, \qquad \left\lfloor \frac{\Pi_s(j)}{n_t} \right\rfloor \neq \left\lfloor \frac{\Pi_s(j+i)}{n_t} \right\rfloor \tag{56}$$

Each of the n_t sub-frames V_i is interleaved into V_i':

$$0 \le i < n_t, \quad 0 \le j < L_{\mathcal{C}} N_{\mathcal{C}}/n_t, \qquad V_i' \left(\Pi_s(j) \right) = V_i(j) \qquad (57)$$

Then, a new sub-frame V_i'' is built from V_i' as follows:

$$0 \le i < n_t, \quad 0 \le j_1 < L_{\mathcal{C}} N_{\mathcal{C}}/n_t^2, \quad 0 \le j_2 < n_t, \qquad V_i'' \left((i + j_2) \bmod n_t + j_1 n_t \right) = V_i' \left(j_2 + j_1 n_t \right)$$

$$(58)$$

The above construction keeps blocks of n_t bits of V_i' in positions corresponding to the same n_t time periods in V_i'', but with a cyclic shift of i positions in a block of size n_t.

6.2   Basic interleaver construction

Let us generalize the interleaver construction to design a basic interleaver $\mathcal{I}_{N_I, S_I, L_I}$ for N_I channel inputs, a frame size S_I bits and a separation L_I. We described $\mathcal{I}_{n_t, L_{\mathcal{C}} N_{\mathcal{C}}, L_I}$ in the previous section. For more general system configurations, the basic interleaver $\mathcal{I}_{N_I, S_I, L_I}$ will be used in the sequel.

In Fig. 6, we present the basic interleaver for $N_I = 4$ channel inputs. Codeword bits are distinguished by four different patterns, each pattern corresponding to a specific channel input. In step 1, the codeword is demultiplexed into N_I sub-frames V_i, $i = 0, \ldots, N_I - 1$, of length S_I/N_I each, as presented in the previous section. In step 2, each vector V_i of size S_I/N_I is interleaved by the S-random-like interleaver into a vector V_i'. In step 3, we build a $N_I \times S_I/N_I$ matrix as the concatenation of S_I/N_I^2 matrices of size $N_I \times N_I$. The latter are circulant matrices where the first row contains the N_I first values of V_0', and the second row contains the first N_I values of V_1'. Rows 3 and 4 are built from V_2' and V_3' similarly.

Finally, the $N_I \times S_I/N_I$ matrix is transmitted over the space-time channel by distributing its rows on channel inputs and its columns on time periods.

This interleaving guarantees that $(L_I - 1)N_I + 1$ consecutive codeword bits are not transmitted

during the same time period. The value of L_I of the S-random-like interleaver should be chosen as large as possible in order to take into account long error events. An upper bound for L_I can be found based on the interleaver separation similar to classical S-random [16]. The interleaver has a sliding input separation equal to L_I and an output block separation equal to N_I within a sub-frame. Hence, drawing a a simple two-level tree representation would lead to $(2L_I - 1) \times N_I \leq S_I/N_I$, rewritten as

$$L_I \leq \frac{1}{2} \left(\frac{S_I}{N_I^2} + 1 \right) \tag{59}$$

6.3   Interleaver design for quasi-static MIMO channels with M-ary input

In section 6.1, we have presented an interleaver for MIMO quasi-static channels and BPSK modulation. For a modulation with higher spectral efficiency, erroneous bits in an error path should be dispatched on different time periods and equally transmitted over all the transmit antennas and bit positions. Repartition on different bit positions is required as different bits of a modulation scheme are not equally protected. These conditions are satisfied by the $\mathcal{I}_{mn_t, L_c N_c, L_I}$ interleaver.

Increasing the diversity by transmitting erroneous bits on all antennas is more important than increasing the coding gain by transmitting them on all modulation bits. The n_t first sub-frames should be transmitted on the n_t transmit antennas and on the first mapping bit. The second block of n_t sub-frames should be transmitted on the second mapping bit, and so on.

6.4   Application to linear precoding

When a linear precoder is used to recover a part of the transmit diversity, the new channel matrix SH has $sn_t \times sn_t$ rows and columns. Linear precoders have been optimized in section 5 when at most one erroneous bit is observed on each precoding time period. We have shown that the precoded channel output is divided into independent blocks, we modify the order of the rows as

follows ($s' = s/n_s$ and $N'_t = s's$)

$$\forall l_2 \in [1, n_t/s'], \forall l_1 \in [1, ss'], \forall t \in [1, n_s], \forall i \in [1, s'], \forall v \in [1, s'],$$

$$S_{(l_1-1)n_t/s'+l_2, v+(l_2-1)n_t/s'+(i-1)n_t+(t-1)s'n_t} =$$

$$\frac{1}{\sqrt{N'_t}} \exp\left(2j\pi \left[(l_1-1)\left(\frac{1}{\Phi^{-1}(2N'_t)} + \frac{v-1+(i-1)s'+(t-1)s'^2}{N'_t}\right) + (i-1)\left(\frac{1}{\Phi^{-1}(2s')} + \frac{v-1}{s'}\right)\right]\right) \quad (60)$$

and 0 elsewhere.

Now, the n_t/s' consecutive rows of S lead to independent row vectors $S_l H_k$ that look like a true multiple antenna channel. In this case, the interleaver $\mathcal{I}_{smn_t, L_{\mathcal{C}}N_{\mathcal{C}}, L_I}$ is designed for diversity and gain exploitation. As presented in the previous subsection, the sn_t first rows of the last interleaver matrix will be transmitted on the first mapping bit, and so on.

6.5 Interleaver design for block-fading MIMO channels

For block-fading channels, n_c different channel realizations occur during the codeword. In order to take advantage of the transmission and time diversity given by the linear precoding and the n_c different realizations of a block-fading MIMO channel, the interleaver of a BICM should place consecutive bits on different precoding time periods and equally distribute them among all linear precoding rows and all n_c channel realizations.

We extract n_c sub-frames from the codeword, each sub-frame will be transmitted on one of the n_c blocks, and only experience one channel realization. We interleave each sub-frame with the interleaver optimized for MIMO quasi-static channel to exploit the linear precoding diversity. The demultiplexing into n_c sub-frames is done in the same manner as for the channel inputs in step 1 of Fig. 6:

$$0 \le i_{n_c} < n_c, 0 \le j < L_{\mathcal{C}} N_{\mathcal{C}}/(n_c n_t), V_i^{n_c}(j) = V\left((i_{n_c} + j) \bmod n_c + jn_c\right) \quad (61)$$

This demultiplexing/interleaving is sufficient to exploit the time diversity. Indeed, there is no interference between the symbols experiencing the different channel realizations contrary to symbols

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transmitted on different linear precoding rows and bit positions.

6.6 Application to turbo-codes

The BICM precoder and interleaver have been designed to provide full diversity and optimal coding gain for any pairwise error probability. However, the error rate is given by the probability to leave the Voronoi region. With convolutional codes, the number of neighbors increases with the frame length whereas the minimum Hamming distance d_{Hmin} remains constant. Thus, the frame error rate increases with frame length. To obtain the opposite behavior, the Euclidean distance must increase with frame length and provide a performance gain higher than the performance degradation due to the increased number of neighbors. It has been shown in [24][9] that turbo-like codes can fulfill such a condition over block-fading channels. We use the coding scheme presented in Fig. 7. To the classical parallel turbo-code using two encoders RSC1 and RSC2 and an interleaver Π_t, we add a de-interleaving Π_t^{-1} of coded bits at the output of RSC2. Thanks to this de-interleaving, error events are localized and the optimized channel interleaver can be applied.

7 Simulation results

In this section, we evaluate the performance of actual iterative joint detection and decoding of the ST-BICM. The APP detector is performed by exhaustive marginalization. The set of 2^{mN_t} noiseless received precoded symbols zSH is computed once per channel block realization since the channel matrix SH is constant during the block. This results in a complexity reduction for the marginalization, which now requires around $L_{\mathcal{C}}N_{\mathcal{C}}/(msn_t)2^{mN_t}$ operations per iteration if $s \ll L_{\mathcal{C}}N_{\mathcal{C}}$. Reduced complexity quasi-optimal or sub-optimal MIMO detectors could also be used, e.g., a SISO list sphere decoder [27][37][3][7], a SISO-MMSE detector [17][40] or a detector using sequential Monte Carlo method [15].

Let us consider a 2×1 quasi-static ($n_c = 1$) MIMO channel and QPSK modulation. We use

$(7,5)_8$ NRNSC or $(3,2)_8$ NRNSC codes with rate $1/2$ and a blocklength of 1024 coded bits. From the Singleton bound we know that full diversity can be achieved without linear precoding. We compare on Fig. 8 the performance obtained with a classical PR interleaver and the performance obtained with the optimized interleaver described in section 6. Full diversity order is only achieved with the optimized interleaver, for which the performance slope is equal to the one of the outage probability. The optimized interleaver provides performance improvement without any increase of complexity neither at the transmitter nor at the receiver. In most cases, the PR interleaver only provides a diversity n_r, i.e., it does not allow any transmit diversity order recovery. The $(7,5)_8$ NRNSC code achieves a higher coding gain than the $(3,2)_8$ NRNSC code. It achieves performance within only 2.5 dB from the outage capacity with Gaussian input and within 1.5 dB from the outage capacity with QPSK input. The performance lower bound corresponding to ideally pre-coded BICM is also drawn. It is obtained from the performance of the same coded modulation transmitted on a $1 \times n_c n_t n_r$ SIMO channel, as explained in section 5.1. There is a 1 dB gap between ideal and actual performances with the $(3,2)_8$ NRNSC code and a 0.75 dB gap with the more powerful $(7,5)_8$ NRNSC code. This confirms the analytical result of section 5.1 obtained for ML performance: The higher the Hamming weight is, the closer to the ideal performance the actual iterative receiver can perform. However, a better code does not always provide better frame error rate. Indeed, we have seen that, when $w \geq n_t n_c$, the full diversity of the considered pairwise error probability can be achieved with an ideal interleaver. The remaining $w - n_t n_c$ BSK distances are uniformly distributed among all the channel states. A better error correcting code with greater Hamming weights w' does not enhance the diversity but the coding gain per pairwise error probability. However, the degradation induced by the increased number of neighbors may be higher than the improvement brought by increased coding gains. How to handle this trade-off is left for further study.

In Fig. 9, we show the performance of a rate-$1/2$ $(7,5)_8$ NRNSC code over a 2×2 MIMO block-fading channel with $n_c = 2$ and QPSK input. The frame length is 256 coded bits. With a

PR interleaver, a diversity order $n_r = 2$ is achieved, as transmit diversity is not collected. Even with the optimized interleaver, full diversity is not obtained at the last iteration. Indeed, the Singleton bound is equal to 6 without linear precoding. Two different linear precoders, the Golden code and the DNA code, both with $s = 2$, are used to achieve the full diversity order 8. The slope difference between diversity orders 6 and 8 is not significant. However, linear precoding provides an additional coding gain which allows to perform within 2 dB from the outage capacity with Gaussian input using a four-state convolutional code and a small frame length. The Golden code does not satisfy the equal norm condition, which induces a slight loss in coding gain. Neverthe-less, this loss is fully compensated by the averaging of the $d_{k,l,i}$ into equal $\gamma_{k,l}$ values provided by the error correcting code as explained in 5.1. For a higher frame length, the performance with convolutional codes is degraded. Therefore, we will also investigate performance with turbo-codes.

In Fig. 10, we compare two strategies for achieving full diversity with BICM: linear precoding and constellation expansion [25]. Constellation expansion consists in increasing m while decreasing the coding rate, in order to achieve the full diversity without precoding and with the same spectral efficiency. A MIMO 2×2 channel with $n_c = 2$ is considered. The frame length is 1024 coded bits. Using QPSK modulation and rate-1/2 coding, full diversity is not achieved. Using a precoded QPSK with $s = 2$ and a 16-state rate-1/2 $(23, 35)_8$ NRNSC code having minimal Hamming distance 7, we get the same spectral efficiency, 2 bits per channel use, and the Singleton bound is equal to 8, the full diversity order. We compare this full-diversity scheme using linear precoding with a scheme using constellation expansion from QPSK to 16-QAM with a 64-state rate-1/4 NRCSC code having generator polynomials $(135, 135, 147, 163)_8$ and minimal Hamming distance 20. With the latter scheme, we get the same spectral efficiency and the Singleton bound is also equal to 8. The linear precoder provides a greater diversity order at the first iteration. At the last iteration, both schemes have same diversity and the precoded scheme slightly outperforms the scheme with constellation expansion. Since the detector complexity is around $L_\mathcal{C} N_\mathcal{C}/(msn_t)2^{mN_t}$ operations per iteration if $s \ll L_\mathcal{C} N_\mathcal{C}$, the detection of the precoded system is as complex as

the detection of the one with constellation expansion. However, channel decoding of the 64-state $(135, 135, 147, 163)_8$ NRNSC code is more complex than the decoding of the 16-state $(23, 35)_8$ NRNSC code. Thus, to get a same performance, it is less complex to use linear precoding than to use constellation expansion. When choosing a 64-state NRNSC $(133, 171)_8$ code with rate $1/2$ and minimal Hamming distance 10, the coding gain is increased by almost 1 dB.

In order to increase the frame length without degrading performance, we now consider turbo-codes. Fig. 11 illustrates the performance of a $(7, 5)_8$ RSC turbo-code over a 1×1 channel with $n_c = 4$, QPSK input and either a PR or an optimized interleaver. Two different frame lengths (256 and 2048 coded bits) are tested. With the PR interleaver and without precoding, the full diversity order 4 is not achieved. If the optimized interleaver is used, the full diversity order is not achieved neither, but the smaller slope is not visible down to a FER equal to 10^{-3}. A similar behavior is obtained with PR interleaver and precoding $s = 2$. Finally, the DNA precoded modulation with optimized interleaver achieves full diversity performance within less than 2 dB from the outage capacity with Gaussian input.

Fig. 12 illustrates the performance of a $(7, 5)_8$ RSC turbo-code over a 2×2 quasi-static channel with QPSK input and either a PR or an optimized interleaver. Two different frame lengths (256 and 2048 coded bits) are tested. With the PR interleaver, the full diversity order 4 is not achieved, and the performance degrades when the frame length increases, as with convolutional codes. With the optimized interleaver, the full diversity order is achieved and the frame error rate decreases when the frame length increases. The system using DNA precoding ($s = 2$), optimized interleaver and a turbo code finally performs within 1 dB from the outage capacity with Gaussian input.

Fig. 13 represents the performance of a $(7, 5)_8$ RSC turbo-code over a 4×1 quasi-static channel with BPSK input and either a PR or an optimized interleaver. Two different frame lengths (256 and 2048 coded bits) are tested. Without linear precoder and using a PR interleaver, the

full diversity gain is not achieved. Asymptotically, the observed diversity is $n_r = 1$, but, for low E_b/N_0, the performance is close to the performance obtained with the optimized interleaver. Indeed, the turbo-code generates a large amount of errors for low E_b/N_0 and the probability of satisfying the ideal interleaving condition with a PR interleaver is high. However, when E_b/N_0 is high, only neighbors have an influence on the error rate and it is crucial to place the few erroneous bits on all the channel states. This behavior is stressed with increased frame length. To achieve maximum diversity, according to the Singleton bound, a precoding with at least $s = 2$ is needed. This is confirmed by the simulation results and again the error rate decreases when the frame length increases. With the 4×1 MIMO channel, a large amount of interference exists between the transmit antennas. Nevertheless, performance is within 2.5 dB from the outage probability with Gaussian input. Performance will be even closer to the outage probability with a higher number of receive antennas or channel realizations.

On Fig. 14, performances of NRNSC codes and parallel turbo-codes with RSC constituent codes over a 2×1 quasi-static MIMO channel are drawn versus frame size for $E_b/N_0 = 15$ dB. Performance of the Alamouti scheme [1] having same spectral efficiency without channel coding is also drawn as a reference. The frame error rate increases with the frame size when using Alamouti scheme or NRNSC codes whereas it remains constant when using turbo codes. This strong property may be in part explained by the interleaving gain of the turbo-code but further research is required on this point.

8   Conclusions

In this paper, we have analyzed the ideal behavior of an ST-BICM using full-rate linear precoding on a MIMO block-fading channel. Ideal performance has been derived analytically using exact pairwise error probabilities under ideal interleaving conditions. Using a bound on the diversity

order, we have shown how to set the time dimension of the linear precoder. Then, we have presented how to design the linear precoder and the interleaver to obtain an ST-BICM achieving full-diversity and performing close to the ideal performance and the outage probability. Fig. 15 summarizes the optimization steps followed in this paper. The proposed DNA precoder slightly outperforms the algebraic Golden code. Furthermore, the design of DNA precoders holds for any parameter set (n_t, s, n_s), whereas algebraic codes have to be specifically designed for each pair (n_t, s). We have also shown that, for a same performance, using linear precoding is less complex than using constellation expansion. Finally, using turbo codes with the optimized interleaver, we have obtained an FER which does not increase with the frame length.

Acknowledgment

A Derivation of $E_{H_k}\left[\Psi_{\boldsymbol{LLR}_k}(j\nu)\right]$ for block-fading channels

We first consider $n_s = 1$ and extend the result to any value of n_s.

A.1 Precoding matrix experiences one channel realization $(n_s = 1)$

For $n_s = 1$, the quasi-static channel matrix H_k is defined as $H_k = \text{diag}\left(H_k^{[1]}, \ldots, H_k^{[1]}\right)$, $H_k^{[1]}$ being repeated s times. From S_l, we construct the $s \times n_t$ matrix $S_l'^{[1]} = \left(S_l^{[1][1]\ T}, S_l^{[1][2]\ T}, \ldots, S_l^{[1][s]\ T}\right)^T$. The row vector $S_l^{[1][i]}$ of size n_t denotes the i-th sub-part of S_l. The n_r columns h_i of $H_k^{[1]}$ are independent realizations of an $n_t \times 1$ multiple-input single-output channel. Let us define $M_k^{[1]}$ as an $n_t \times n_t$ Hermitian square root matrix of $\Sigma_k^{[1]} = \sum_{l=1}^{N_t} \gamma_{k,l}^2 S_l'^{[1]*} S_l'^{[1]}$. Thus,

$$M_k^{[1]} = M_k^{[1]*} = U^* \sqrt{\Phi_{\Sigma_k^{[1]}}} U \tag{62}$$

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where $\Phi_{\Sigma_k^{[1]}} = \mathrm{diag}(\vartheta_{k,1}^{[1]}, \ldots, \vartheta_{k,n_t}^{[1]})$, $\vartheta_{k,u}^{[1]}$ being the u-th real eigenvalue of $\Sigma_k^{[1]}$, and U is a unitary matrix. We write

$$\sum_{l=1}^{N_t} R_{k,l} = \sum_{i=1}^{n_r} \sum_{l=1}^{N_t} \gamma_{k,l}^2 h_i^* S'^{[1]*}_l S'^{[1]}_l h_i = \mathrm{Tr}\left(\sum_{i=1}^{n_r} M_k^{[1]} h_i h_i^* M_k^{[1]*}\right) \tag{63}$$

The random variable $\sum_{i=1}^{n_r} M_k^{[1]} h_i h_i^* M_k^{[1]}$ has a Wishart distribution with n_r degrees of freedom and parameter matrix $\Sigma_k^{[1]}$. The characteristic function of the trace of $\sum_{i=1}^{n_r} M_k^{[1]} h_i h_i^* M_k^{[1]}$ is given in [32]. Finally,

$$\begin{aligned}
E_{H_k}\left[\Psi_{LLR_k}(j\nu)\right] &= E_{H_k}\left[\exp\left(\frac{\nu(j-\nu)}{2}\frac{\mathrm{Tr}(\sum_{i=1}^{n_r} M_k^{[1]} h_i h_i^* M_k^{[1]})}{N_0}\right)\right] \tag{64}\\
&= \left(\det(\Sigma_k^{[1]})\det\left(\Sigma_k^{[1]-1} - \frac{\nu(j-\nu)}{2N_0}I_{n_t}\right)\right)^{-n_r} \tag{65}\\
&= \prod_{u=1}^{n_t}\left(1 - \frac{\nu(j-\nu)}{2N_0}\vartheta_{k,u}^{[1]}\right)^{-n_r} \tag{66}
\end{aligned}$$

A.2 Precoding matrix experiences several channel realizations $(n_s > 1)$

For $n_s > 1$, we first decompose each row S_l into n_s sub-parts of size N_t/n_s, denoted $S_l^{[t]}$. Then, each sub-part $S_l^{[t]}$ is decomposed into s/n_s sub-parts $S_l^{[t][i]}$ of size n_t. As different values of t correspond to independent channel matrices $H_k^{[t]}$, the characteristic functions associated with the sub-parts $S_l^{[t]}$ can be multiplied. Substituting s with s/n_s in the mathematical development presented in section A.1, we directly have

$$E_{H_k}\left[\Psi_{LLR_k}(j\nu)\right] = \prod_{t=1}^{n_s}\prod_{u=1}^{n_t}\left(1 - \frac{\nu(j-\nu)}{2N_0}\vartheta_{k,u}^{[t]}\right)^{-n_r} \tag{67}$$

where $\vartheta_{k,u}^{[t]}$ is the u-th eigenvalue of

$$\Sigma_k^{[t]} = \sum_{l=1}^{N_t} \gamma_{k,l}^2 \sum_{i=1}^{s/n_s} S_l^{[t][i]*} S_l^{[t][i]} \tag{68}$$

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$n_t \backslash s$	1	2	3	4	5	6	7	8
1	**1**							
2	**2**	**2**						
3	2		**3**					
4	3	**4**		**4**				
5	3				**5**			
6	4	4	**6**			**6**		
7	4						**7**	
8	5	6		**8**				**8**

Table 1: Diversity order from modified Singleton bound versus number of transmit antennas n_t and spreading factor s, for $R_\mathcal{C} = 1/2$, $n_r = 1$ and $n_c = 1$.

$n_t \backslash s$	1	2	3	4	5	6	7	8
1	**2**	**2**						
2	3	**4**		**4**				
3	4	4	**6**			**6**		
4	5	6		**8**				**8**
5	6	6			**10**			
6	7	8	9			**12**		
7	8	8					**14**	
8	9	10		12				**16**

Table 2: Diversity order from modified Singleton bound versus number of transmit antennas n_t and spreading factor s, for $R_\mathcal{C} = 1/2$, $n_r = 1$, $n_c = 2$.

	$w = 2$	$w = 3$	$w = 4$	$w = 5$	$w = 6$	$w = 7$	$w = 8$
$n_t = 2$	0.00	0.26	0.00	0.09	0.00	0.05	0.00
$n_t = 3$	/	0.00	0.25	0.21	0.00	0.08	0.08
$n_t = 4$	/	/	0.00	0.22	0.26	0.17	0.00
$n_t = 5$	/	/	/	0.00	0.19	0.26	0.24
$n_t = 6$	/	/	/	/	0.00	0.17	0.25
$n_t = 7$	/	/	/	/	/	0.00	0.15
$n_t = 8$	/	/	/	/	/	/	0.00

Table 3: Best gain in dB to be provided by linear precoding with respect to an unprecoded system, with ideal interleaving and for a given pair of codewords with Hamming distance w and BPSK input.
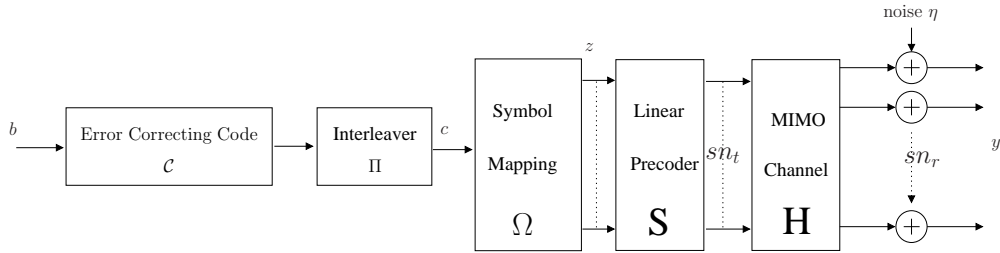
Figure 1: Bit-interleaved coded modulation transmitter and multiple antenna channel model.
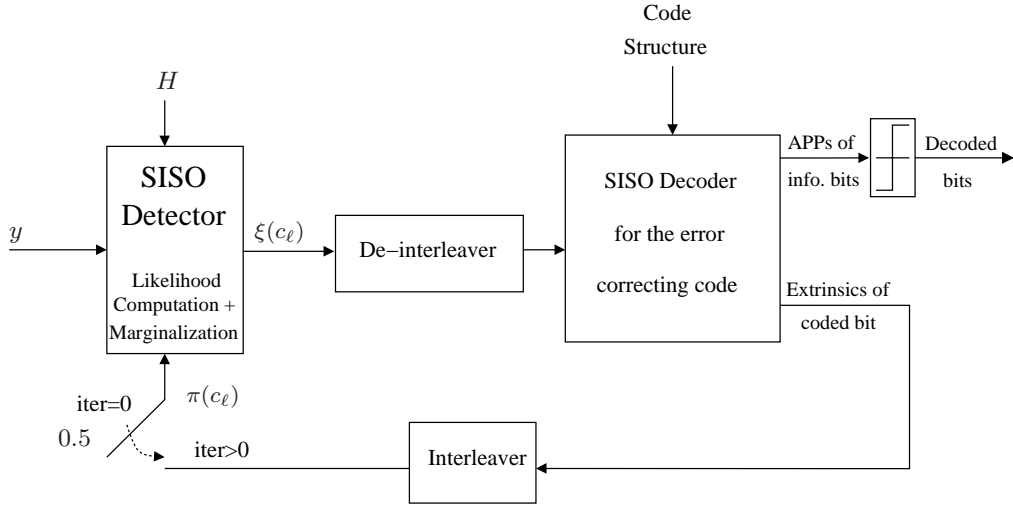
Figure 2: Iterative APP detection and decoding receiver.

Set of time periods T_1 Set of time periods T_2
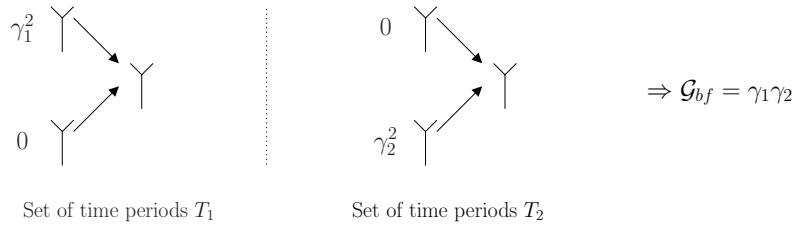
Figure 3: Coding gain for unprecoded 2×1 quasi-static MIMO channel

Set of precoding time periods T_1' Set of precoding time periods T_2'
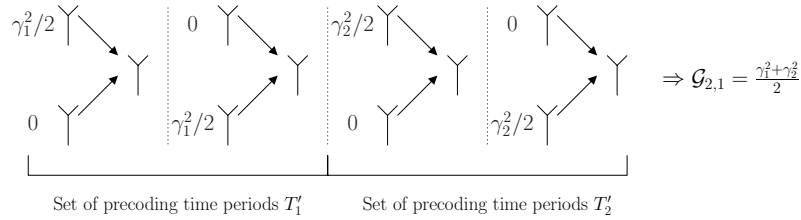
Figure 4: Coding gain for precoded 2×1 quasi-static MIMO channel, $s = 2$
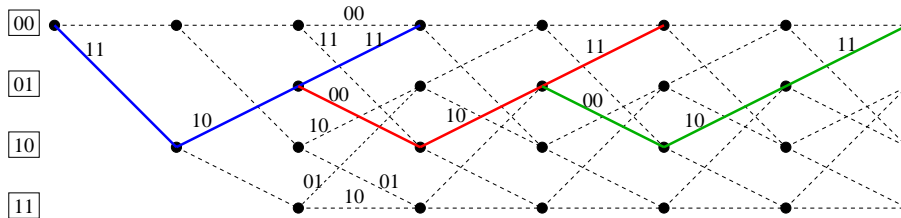
Figure 5: Example of error events with unbalanced Hamming weight when comparing first and second bits on a transition label. The worst unbalance is due to the length-2 cycle between second and third states. Trellis of NRNSC $(7,5)_8$ code.
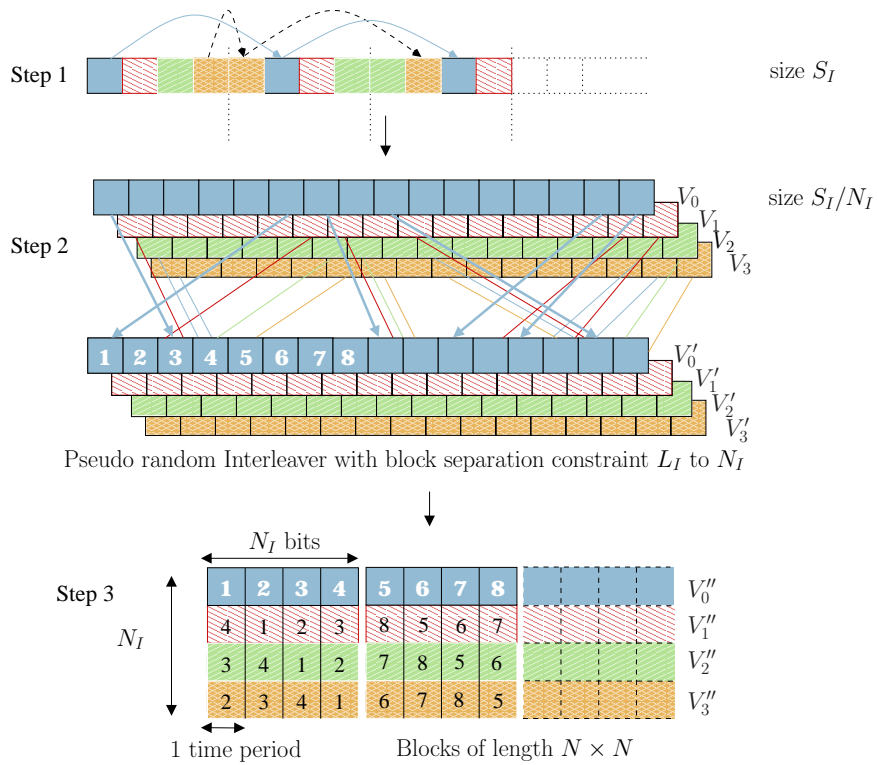
Figure 6: Basic interleaver design for $N_I = 4$ inputs, a frame size S_I and a separation L_I
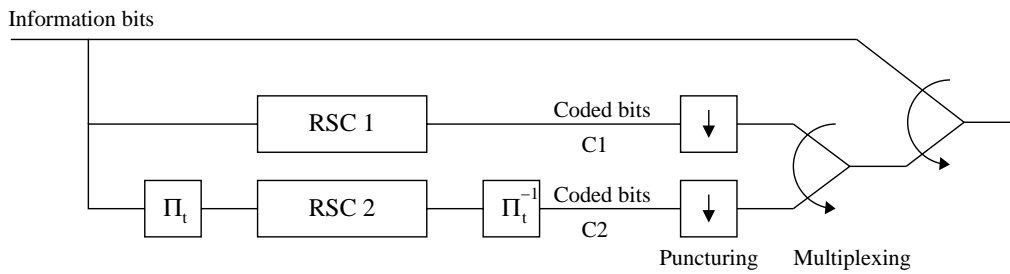
Figure 7: Parallel turbo-code encoder
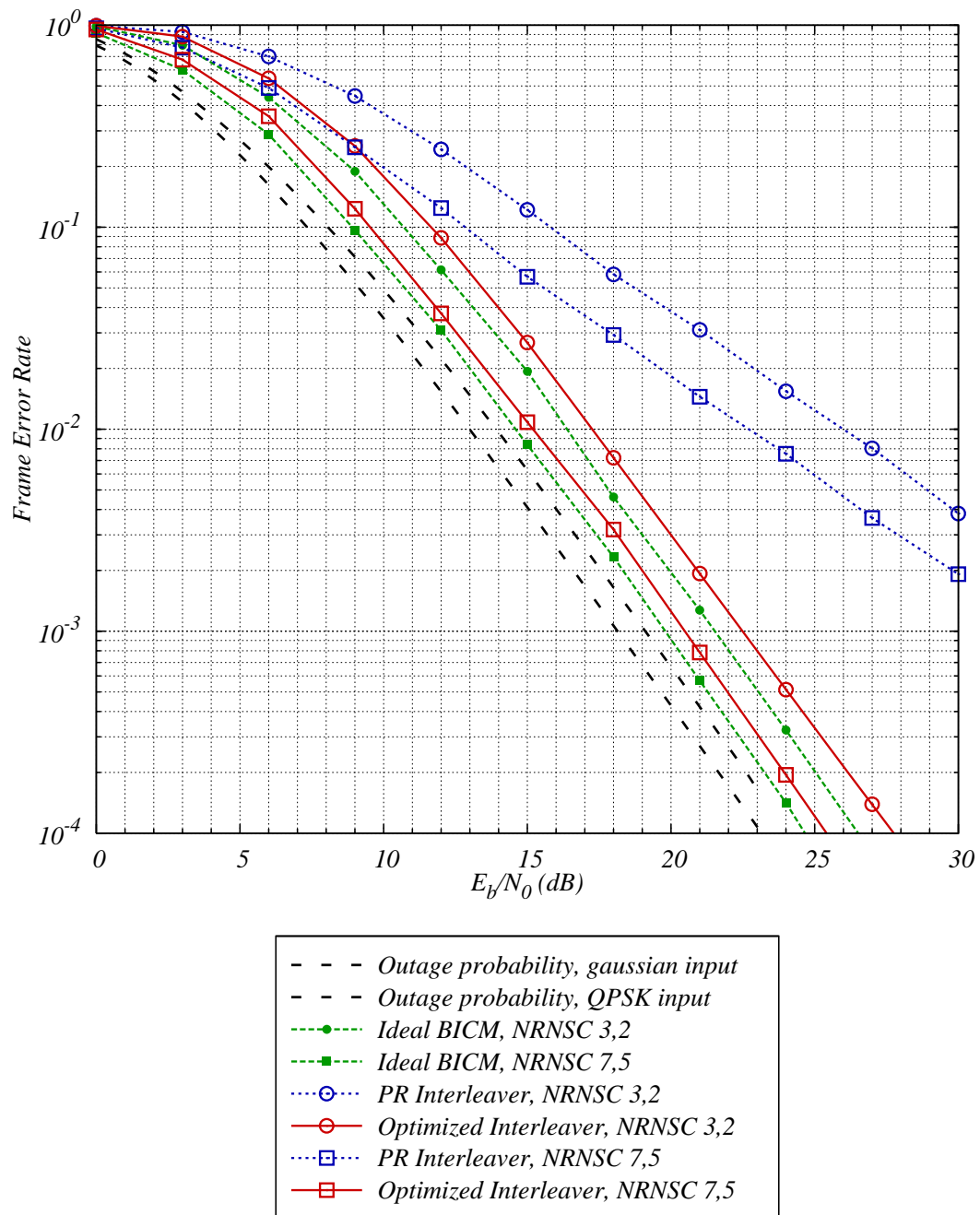
Figure 8: Optimized interleaver with rate-1/2 NRNSC codes - QPSK modulation, 2×1 MIMO channel, $n_c = 1$, 10 iterations, $L_c N_c = 1024$.
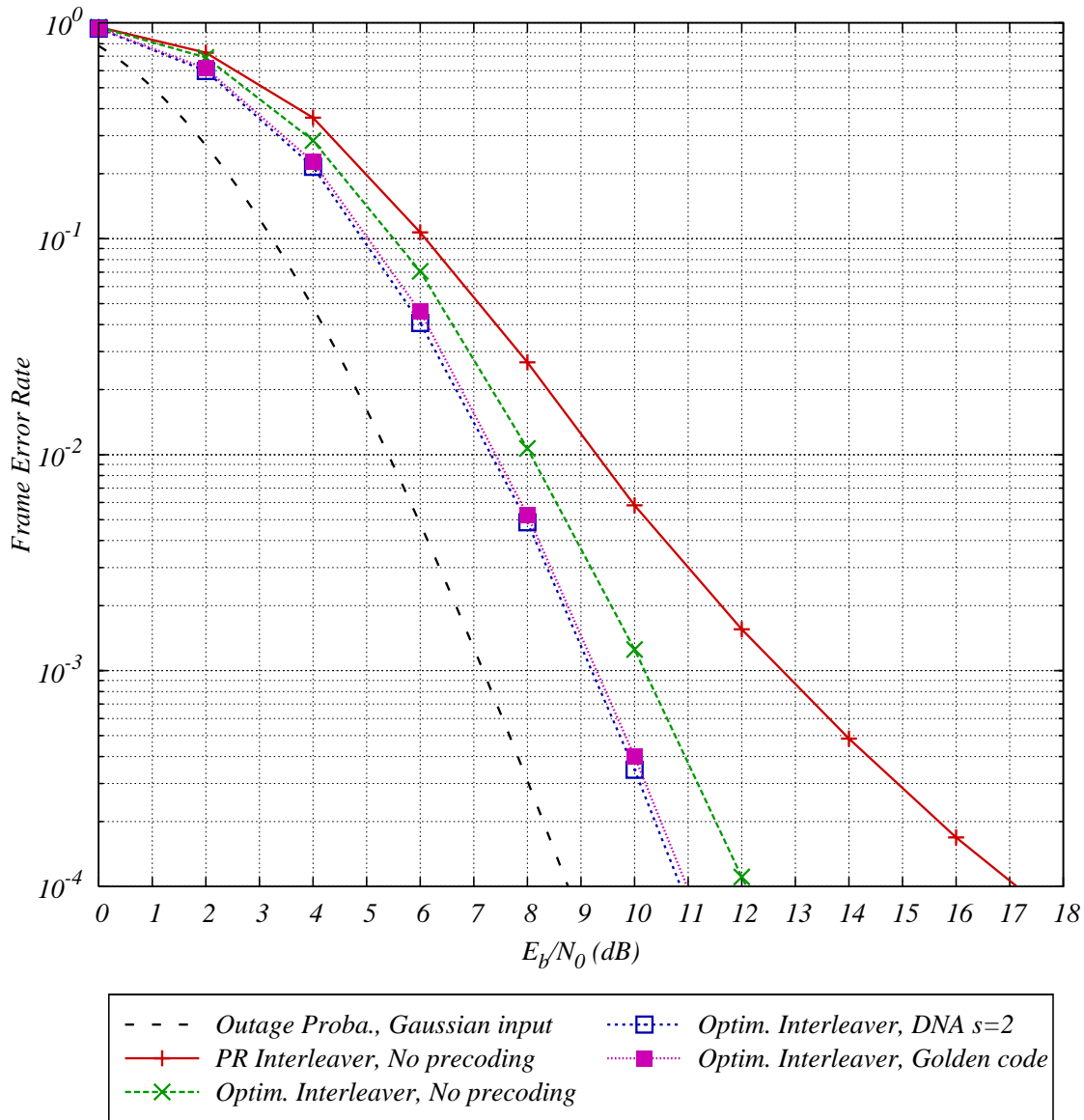
Figure 9: Optimized interleaver with rate-1/2 $(7,5)_8$ NRNSC code and linear precoders - QPSK, 2×2 MIMO channel, $n_c = 2$, 5 iterations, $L_c N_c = 256$ - No linear precoder, DNA cyclotomic precoder $(s = 2, n_s = 1)$, Golden code $(s = 2, n_s = 1)$.
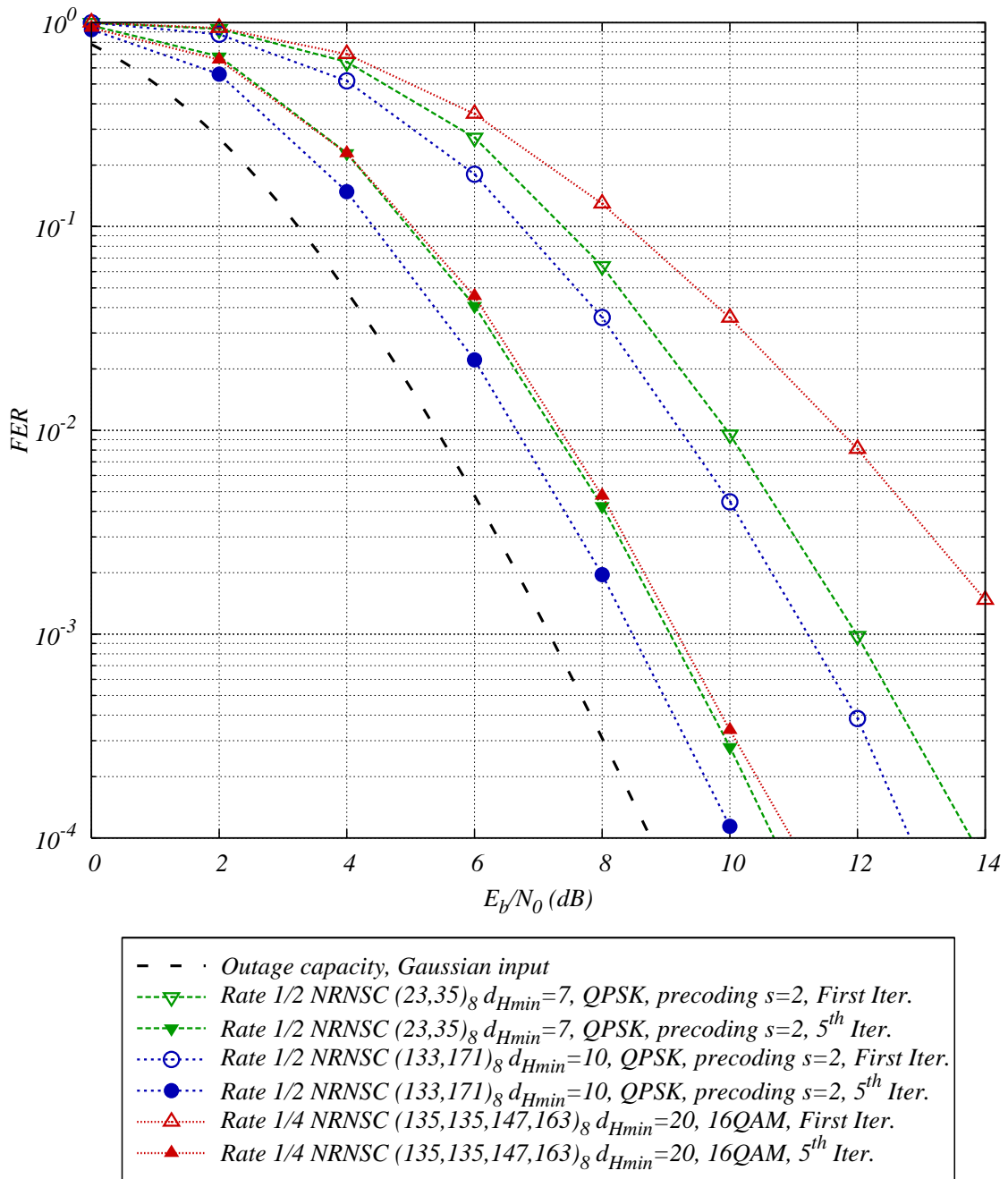
Figure 10: Constellation expansion versus linear precoding - 2×2 MIMO channel, $n_c = 2$, $L_c N_c = 1024$, optimized interleaver.
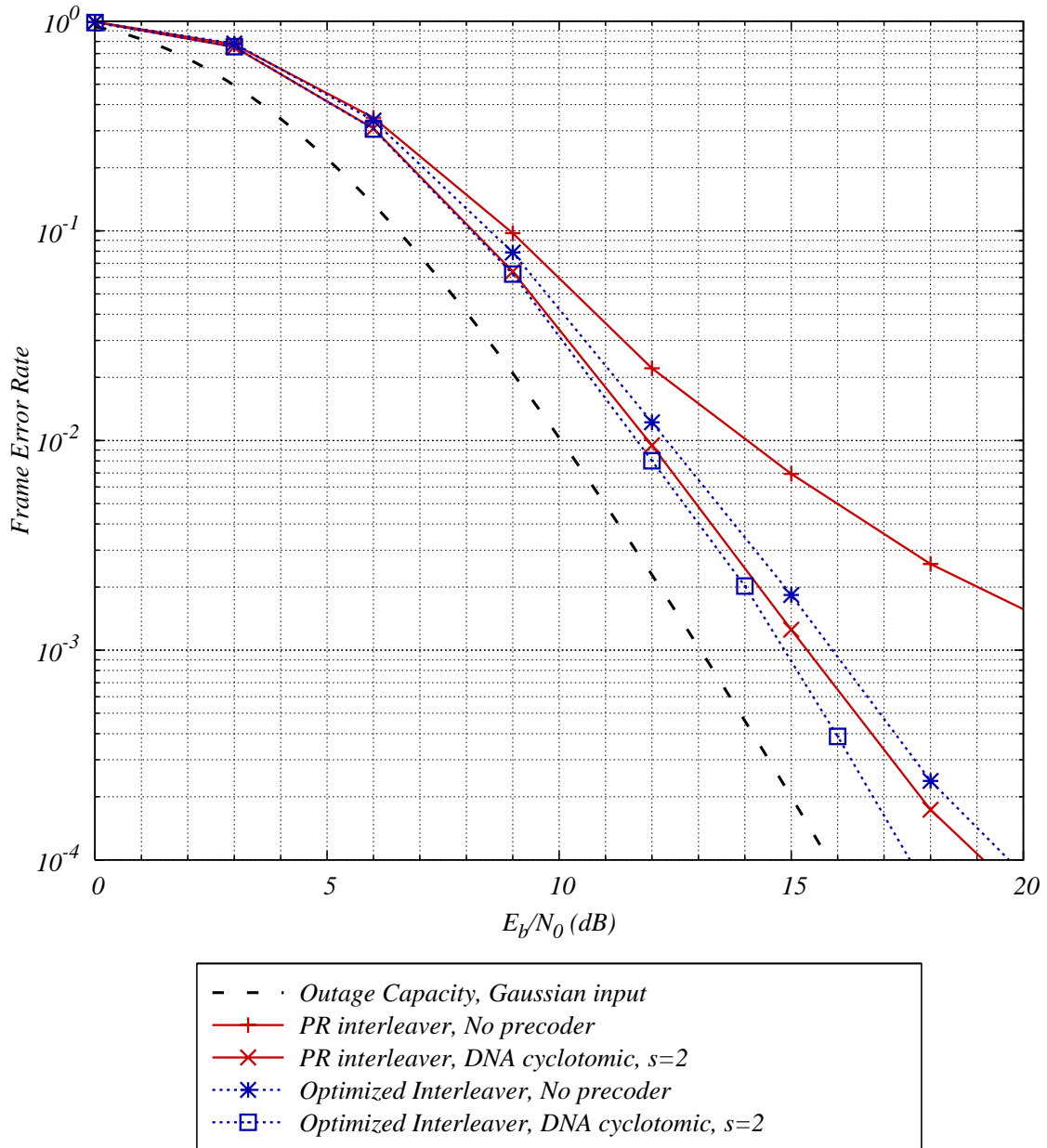
Figure 11: Optimized interleaver with rate-1/2 RSC $(7,5)_8$ turbo-code and DNA cyclotomic precoder - QPSK, 1×1 MIMO channel, $n_c = 4$, 15 iterations, $L_c N_c = 2048$ - Parity check bits of the second constituent are multiplexed via the inverse turbo interleaver.
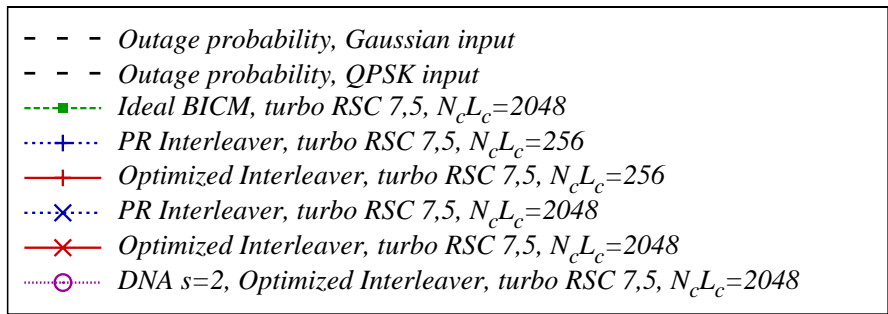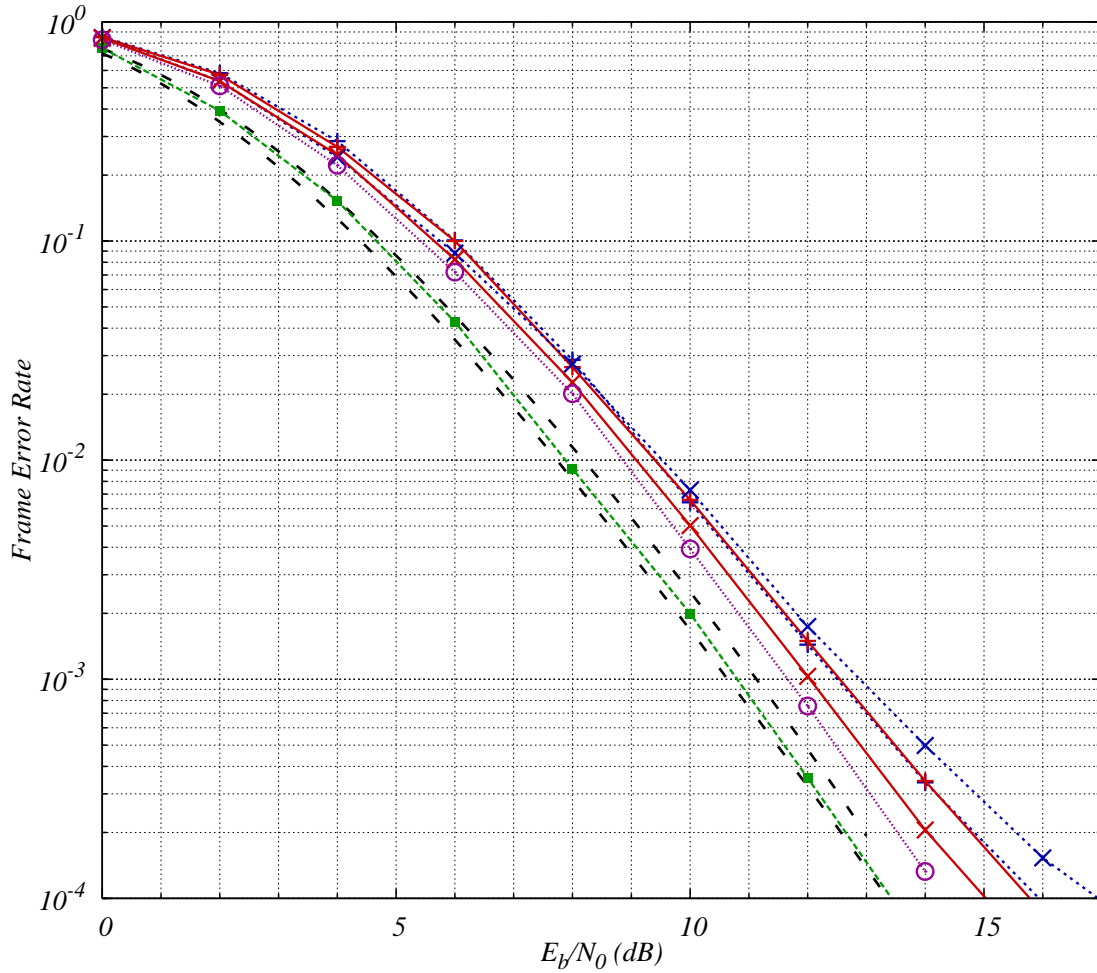
Figure 12: Impact of frame size with a rate-1/2 RSC $(7,5)_8$ turbo-code - QPSK, 2×2 MIMO channel, $n_c = 1$, 15 iterations - Parity check bits of the second constituent are multiplexed via the inverse turbo interleaver.
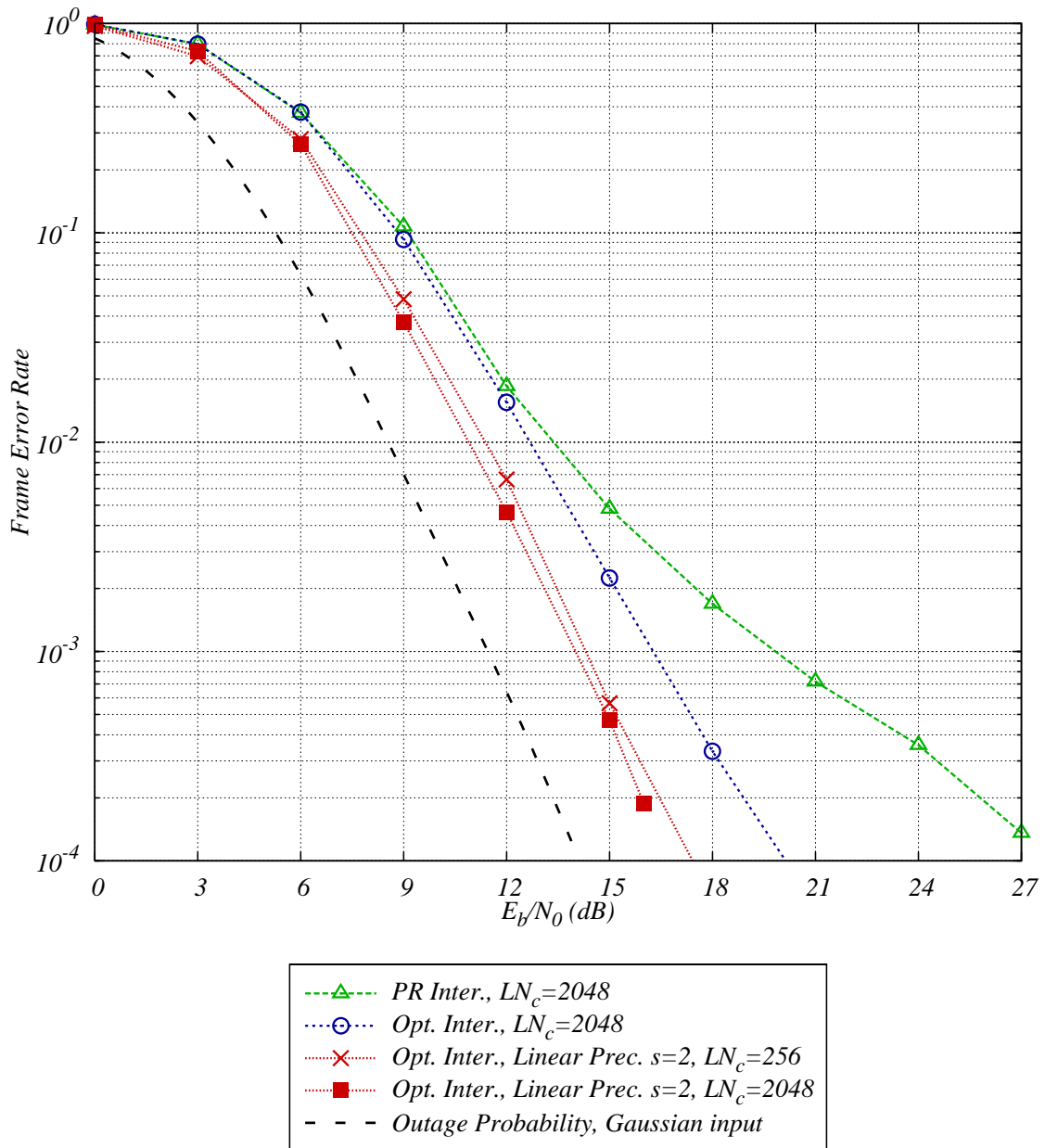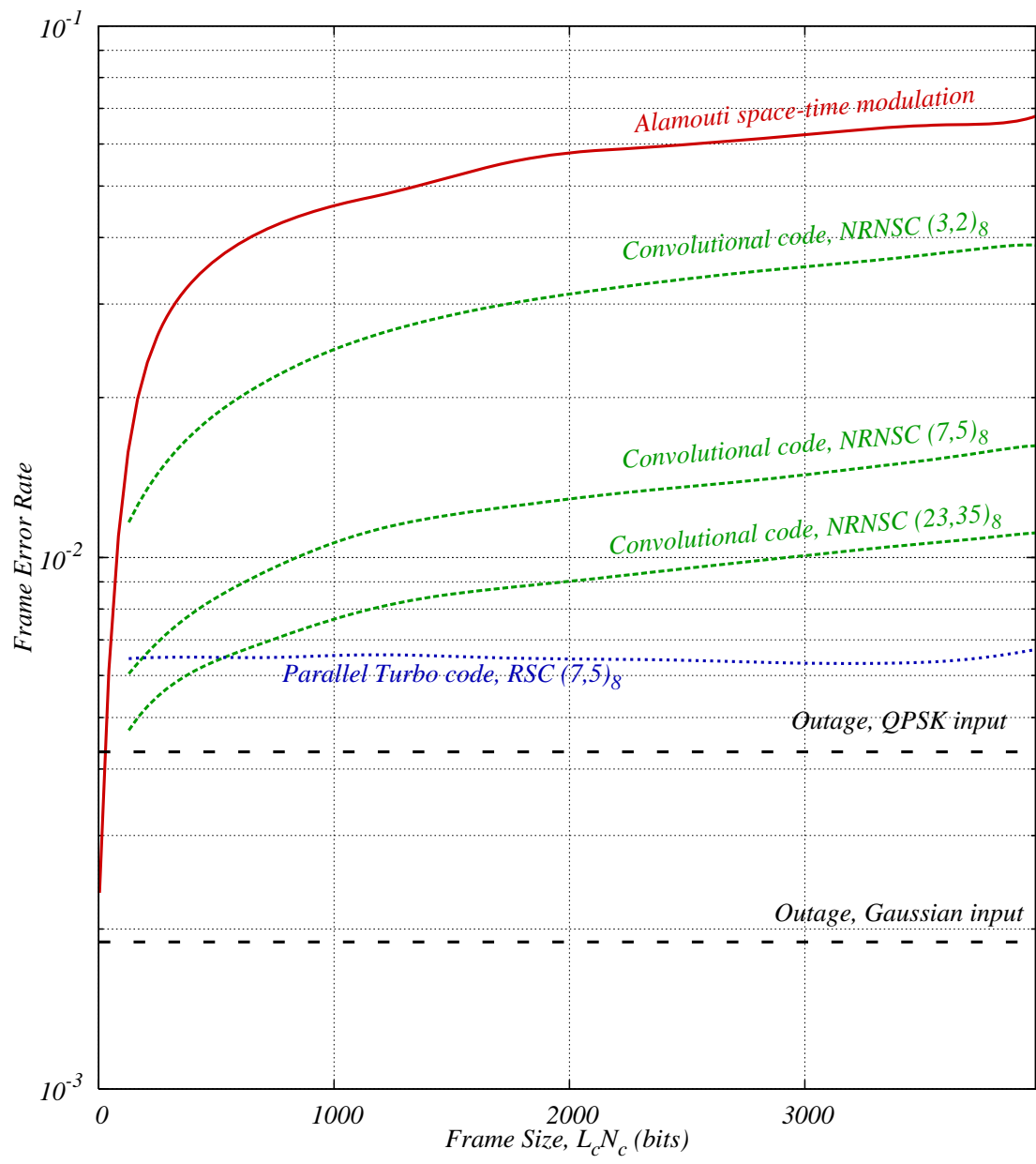
Figure 13: Impact of frame length with a rate-1/2 RSC $(7,5)_8$ turbo-code - BPSK, 4×1 MIMO channel, $n_c = 1$, 15 iterations - Parity check bits of the second constituent are multiplexed via the inverse turbo interleaver.

Figure 14: Frame error rate versus the frame size $L_c N_c$ - BPSK, 2×1 MIMO channel, $n_c = 1$, $E_b/N_0 = 15$ dB - Alamouti STBC, NRNSC codes, Rate one half punctured parallel turbo codes - Parity check bits of the second constituent are multiplexed via the inverse turbo interleaver.
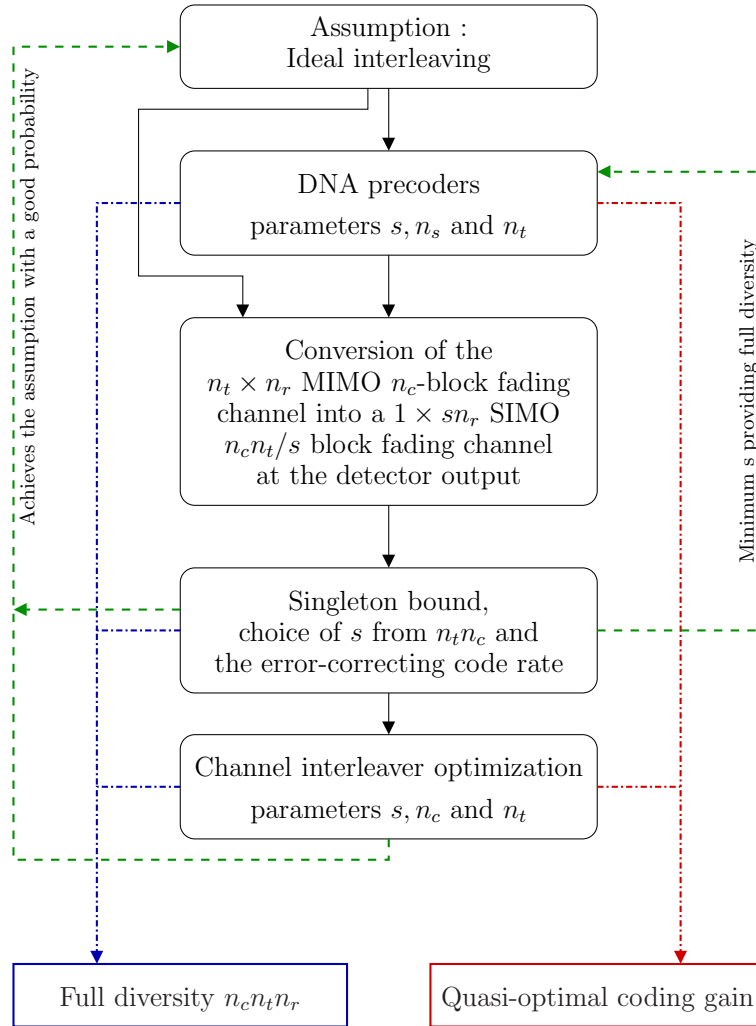
Figure 15: Summary of the space-time BICM optimization process for a $n_t \times n_r$ MIMO n_c-block fading channel. The parameters s and n_s are the time dimension of the $sn_t \times sn_t$ precoding matrix S and the number of independent block channel realizations linked by the precoder respectively.